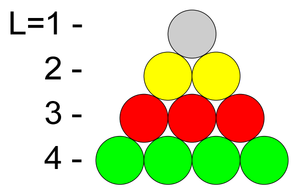
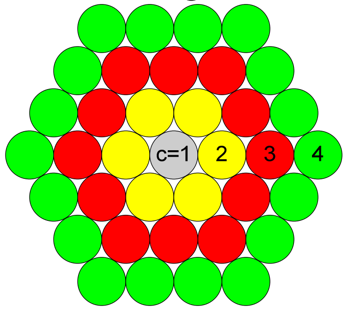
Kepler Conjecture (2 Dimension)

In 1611, Kepler proposed that close packing (either cubic or hexagonal close packing, both of which have maximum densities of $\pi/(3\sqrt{2}) \approx 74.048\%$) is the densest possible sphere packing, and this assertion is known as the Kepler conjecture. Finding the densest (not necessarily periodic) packing of spheres is known as the Kepler problem.

Triangle



Hexagon



$$\#\text{balls} = \frac{L^2 + L}{2} \Rightarrow$$

$$\text{ball area} = \frac{L^2 + L}{2}\pi r^2$$

$$\text{tri area} = \left[\sqrt{3}(L-1)^2 + 6(L-1) + 3\sqrt{3}\right]r^2 \Rightarrow$$

$$\text{ratio} = \frac{\frac{L^2 + L}{2}\pi}{\sqrt{3}(L-1)^2 + 6(L-1) + 3\sqrt{3}} \Rightarrow$$

$$\lim_{L\to\infty} = \frac{\pi}{2\sqrt{3}} = 0.9068996$$

$$\#\text{balls} = 3c^2 - 3c + 1 \Rightarrow$$

$$\text{ball area} = (3c^2 - 3c + 1)\pi r^2$$

$$\text{hex area} = 2\sqrt{3}(4c^2 - 4c + 1)r^2 \Rightarrow$$

$$\text{ratio} = \frac{(3c^2 - 3c + 1)\pi}{2\sqrt{3}(4c^2 - 4c + 1)} \Rightarrow$$

$$\lim_{c\to\infty} = \frac{3\pi}{8\sqrt{3}} = 0.680174$$

$$c = 1 \Rightarrow \text{ratio} = 0.906899682117109$$

$$c = 2 \Rightarrow \text{ratio} = 0.705366419424418$$

$$c = 3 \Rightarrow \text{ratio} = 0.689243758409003$$

$$c = 4 \Rightarrow \text{ratio} = 0.684801800782307$$

$$c = 5 \Rightarrow \text{ratio} = 0.682973834680786$$

$$c = 6 \Rightarrow \text{ratio} = 0.68204852126162$$