Engineering Venue 2007- 09

Probability as Applied to Reliability

September 17, 2007

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AGENDA

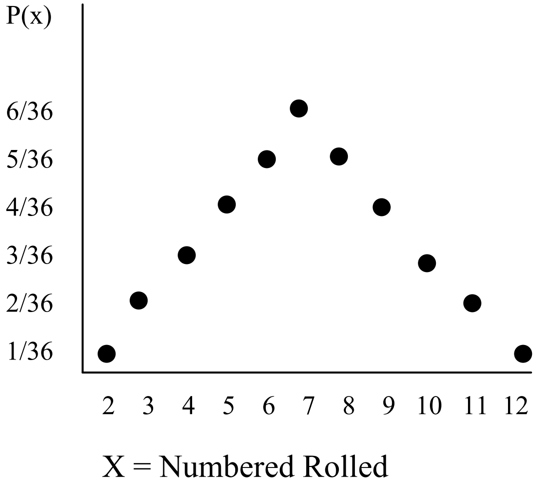
Introduction to Combinatorial Probability and its application to Reliability.

Probability and Dice

- Two Dice – Each 6 Faces – Each Numbered 1 to 6

Roll of Dice

Possible Results	Ways of Getting Result	Relative Frequency
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36



P (x) – Probability of Occurrence = Relative Frequency

- For unbiased dice
- For very large number of rolls (events)

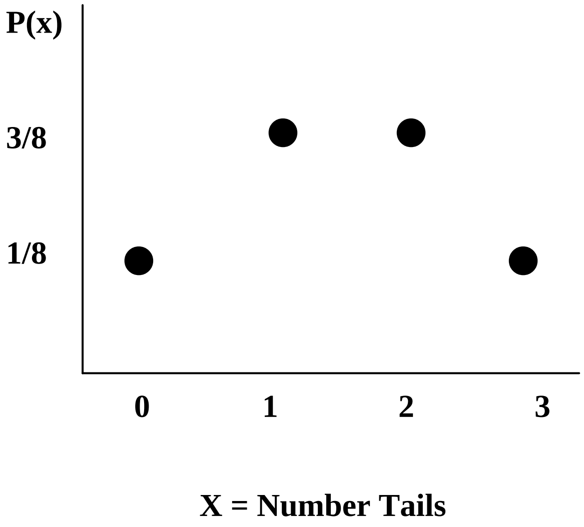
Tails You Lose – Heads I win

- Coin Tossing – Unbiased Coin

  P(H) = P(T) = 0.5

  – Suppose an unbiased coin is to be tossed three times
  – Number of possible events is 8
  – Possible outcomes of each event

Events	Num. Tails	Req. Freq
HHH	0	1/8
HHT	1	
HTH	1	3/8
THH	1	
HTT	2	
THT	2	3/8
TTH	2	
TTT	3	1/8



P (x) = Probability of exactly n (= 0, 1, 2, 3) in 3 tosses of one unbiased coin

Other Probability Expressions

- Bernoulli Trials
  - Random experiment with only two possible outcomes
  - Probabilities of outcomes do not change from trial to trial
  - n trials are independent

    Examples (1) Toss of a coin (head or tail)

    (2) Success or failure of a mission
- Let p = probability of success in some Bernoulli trial

  Then $q = 1 - p$ = probability of failure in the same Bernoulli trial

  Since $1 = p + q$

  Then $1 = (p + q)^n$

Probability of Bernoulli Trials

Let p = Probability of success of a trial

$q = 1 - p$ = Probability of failure of the experiment

n = Number of trials (events)

Using Binomial Expansion (Theorem)

$$1 = (p + q)^n = p^n + \binom{n}{1} p^{n-1} q + \binom{n}{2} p^{n-2}q^2 + \ldots + \binom{n}{m} p^{n-m}q^m + \ldots + q^n$$

Prob n success

Prob (n-1) success

Prob (n-2) success

Prob (n-m) success

Prob (no) success

Prob of at least (n-1) success

Prob of at least (n-2) success

Pascal's Triangle

For Determining Coefficients of Binomial Expansion

$(p + q)^n$, n = 0, 1, 2, …

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

n = row #

k = position #

n = row#									
0	1								
1	1	1							
2	1	2	1						
3	1	3	3	1					
4	1	4	6	4	1				
5	1	5	10	10	5	1			
6	1	6	15	20	15	6	1		
7	1	7	21	35	35	21	7	1	
8	1	8	28	56	70	56	28	8	1
k =	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

Note: Sum of each row = 2^n

Binomial Coefficients

For complex problems with large values of n,

<u>How do we evaluate the coefficients of the Binomial Expansion?</u>

- Four Ways
  - Multiply out expansion algebraically
  - Use expressions $\binom{n}{m} = \frac{n!}{m!(n-m)!} = nCm$
  - Use tables
  - Pascal's Triangle

Coins Revisited

- Assume 3 tosses (trials) are made
- Probability of a head coming is p = 0.5
- Probability of a tail is q = 1 – p = 0.5

$$1 = (p+q)^3$$

$$1 = p^3 + 3p^2q + 3pq^2 + q^3 \text{ or } \binom{3}{0} p^3 + \binom{3}{1} p^2q + \binom{3}{2} pq^2 + \binom{3}{3} q^3$$

$$1 = (.5)^3 + 3(.5)^2(.5) + 3(.5)(.5)^2 + (.5)^3$$

A) P(3H in 3 trials) → $(.5)^3$

B) P(2H in 3 trials) → $3(.5)^2(.5)$

C) P(1H in 3 trials → $3(.5)(.5)^2$

D) P(0H in 3 trials) → $(.5)^3$

Also: P(at least 2H in 3 trials) = P(3H) + P(2H) = $(.5)^3 + 3(.5)^2(.5) = 0.5$

The Link to Reliability

Definition of Reliability

- Reliability is defined as a determination that a **system**, subsystem, unit, or **part** will perform its intended function for a specified interval under certain operational, and environmental conditions.
- Since systems, parts, etc., **do fail**, there is a need to establish Reliability values for them.
- Reliability and Probability are related by the expression, that **Reliability, is equal to Probability of Success**.

$$\mathbf{R = P_s}$$

Use of Binomial Expansion in Reliability

Three (3) identical black boxes are operating "Active Redundant". What is the probability that at least one black box will operate if the reliability (probability of success) of one box is 0.9?

- Probability of success = p = 0.9
- Probability of failure = q = 1 – p = 0.1

$1 = (p + q)^3$

$1 = p^3 + 3p^2q + 3pq^2 + q^3$

$1 = (.9)^3 + 3(.9)^2(.1) + 3(.9)(.1)^2 + (.1)^3$

P (0 failures)
P (1 failure)
P (2 failures)
P (3 failures)

Note: This is a combinatorial calculation and can only be used when all failure rates are the same and not subject to changes.

Use of Binomial Expansion in Reliability cont.

- A generalized solution of this Binomial Probability problem can be described as:
  - n is the total identical components of k of these will fail.
  - $\binom{n}{k}$ ways of k failed components and (n-k) working components
- The probability of occurrence of any particular combination is $p^{n-k}q^k$
- P(exactly k failures out of n) $= \binom{n}{k} p^{n-k} q^k = \frac{n!}{k!(n-k)} p^{n-k} q^k$

Note: This is a combinatorial calculation and can only be used when all failure rates are the same and not subject to changes.

Use of Binomial Expansion in Reliability cont.

What is the probability (P) that two (m) or more engines (in a four engine aircraft) will operate successfully throughout the flight if the reliability of each engine is 0.9?

In this case, $p = 0.9$, $q = 0.1$, $m = 2$, $n = 4$

$$P = \sum_{k=0}^{m} \binom{n}{k} (.9)^{4-k} (.1)^{k}$$

$$= \binom{4}{0} (.9)^4 (.1)^0 + \binom{4}{1} (.9)^3 (.1)^1 + \binom{4}{2} (.9)^2 (.1)^2$$

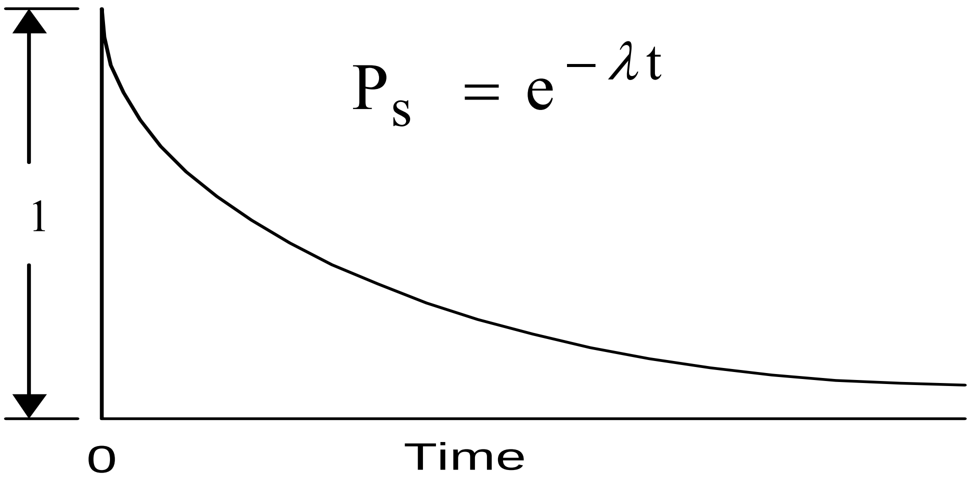
$$= (.9)^4 + 4\,(.9)^3 (.1) + 6\,(.9)^2 (.1)^2 = .9963$$

4 pass → $(.9)^4$
3 pass → $(.9)^3$
2 pass → $(.9)^2$

Note: m + 1 terms

Difference between Constant and Non-constant Failure Rate Devices

Failure Characteristic of a Constant Failure Rate Device



$$P_s = e^{-\lambda t}$$

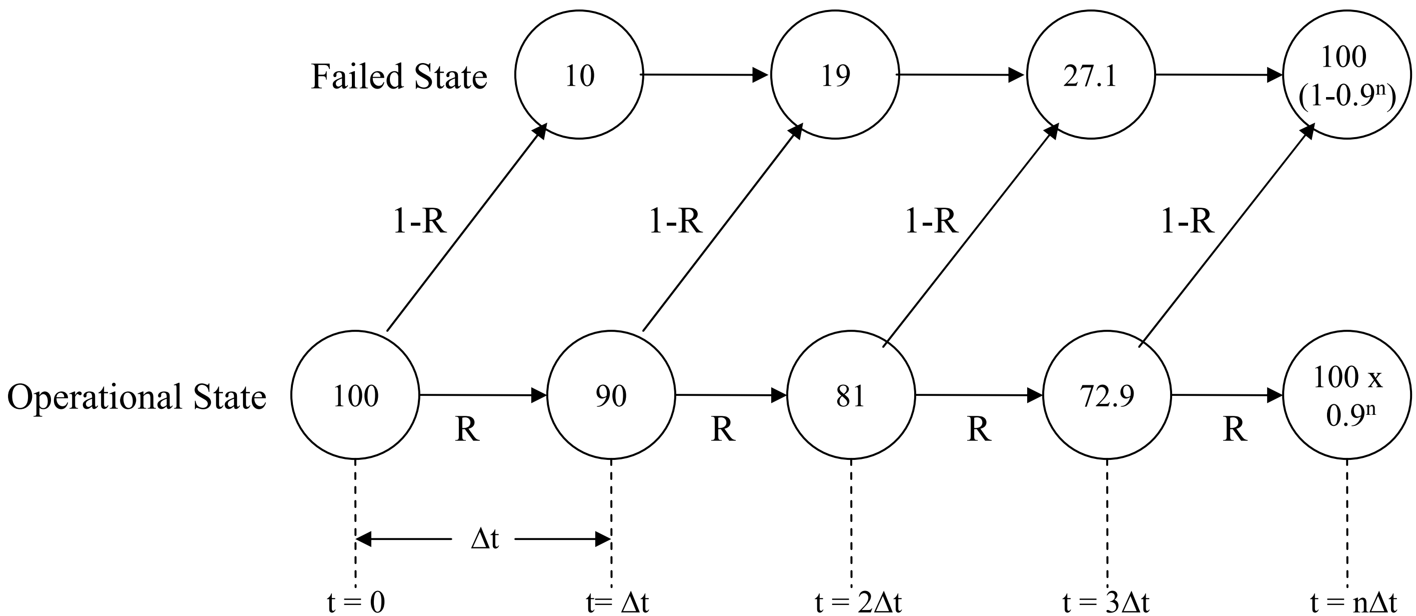
The Reliability or probability of success (Ps) graph of a constant failure rate device is an exponential curve as shown with λ being the constant failure rate, and t is time.

Note:
With respect to constant failure rate devices, Ps = Reliability is a function of the variable time and λ (a constant).

Failure Characteristic of a Constant Failure Rate Device cont.

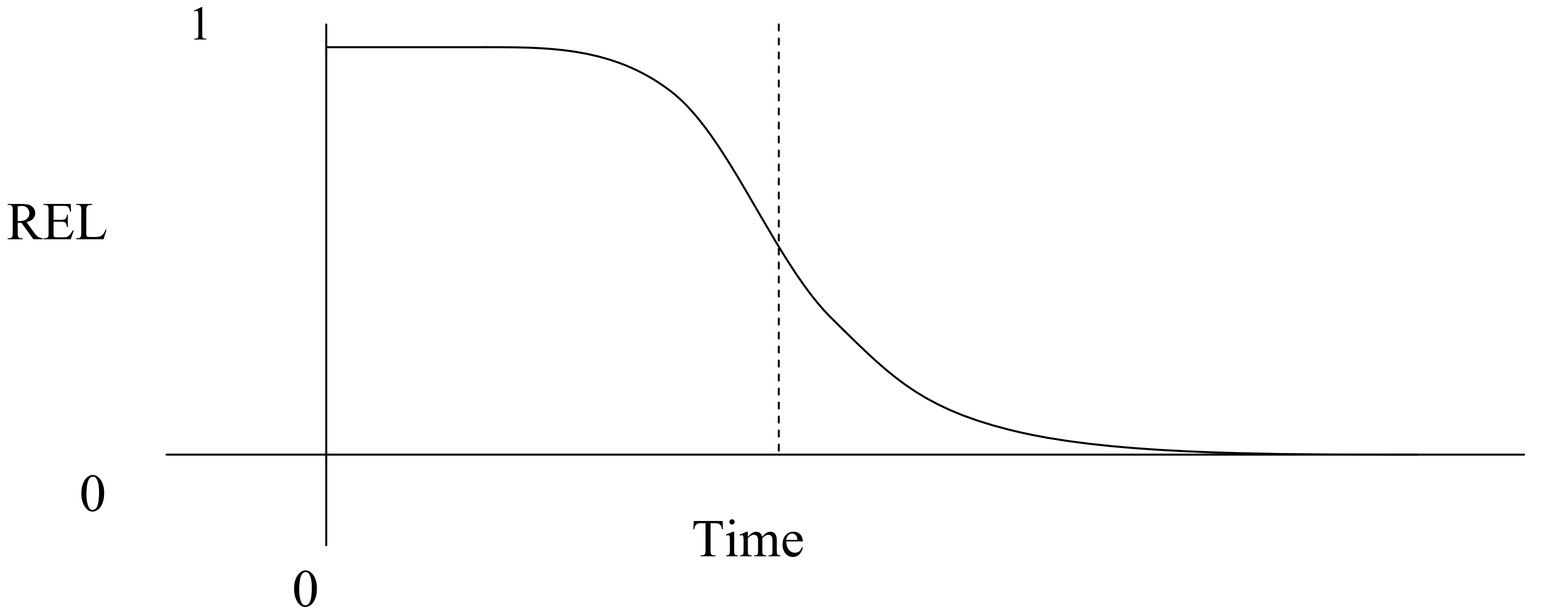
Another way of describing this exponential behavior is as follows:

Assume 100 identical devices initially fully operational, and for a given Δt the Reliability = R = 0.9

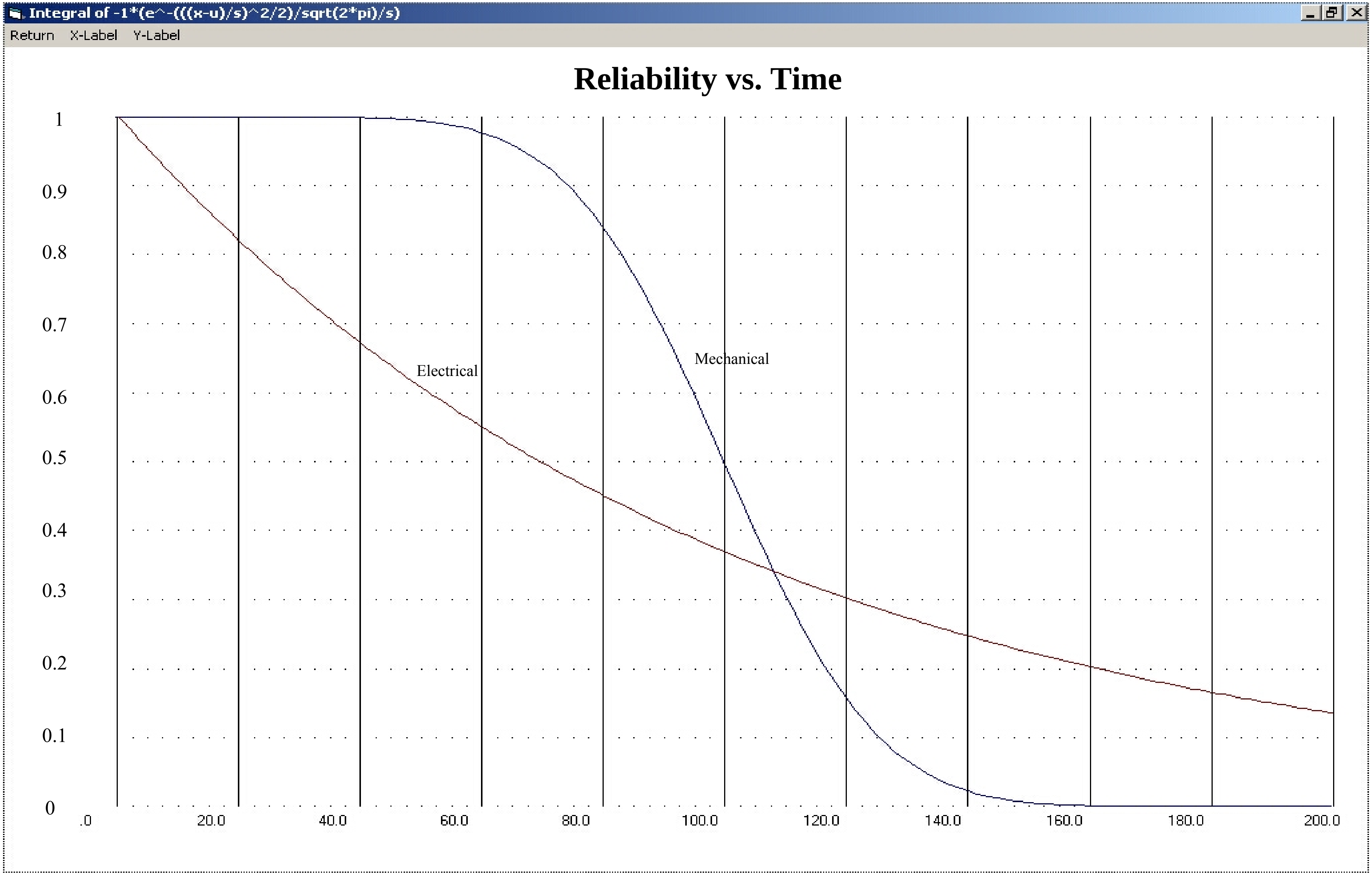


Mechanical Devices

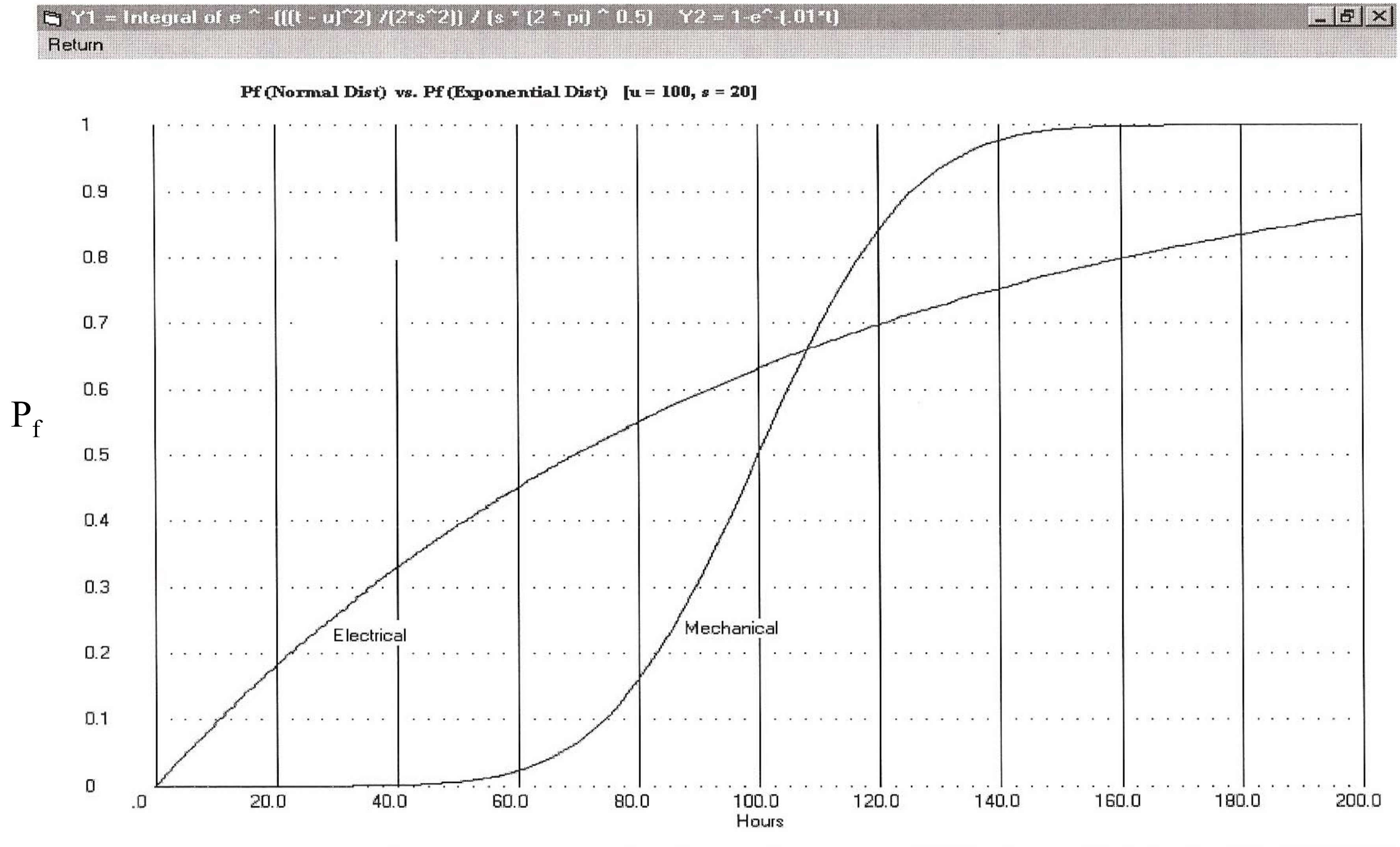
- Mechanical devices exhibit many types of failures including Distortion, Stress/Fracture, Wear and Corrosion.
- For example, excessive vibration (mode of failure), from loss of lubricant (cause of failure), due to distortion (mechanism of failure)
- The reliability characteristic of a typical mechanical device is shown here:



Graph comparing Rel of Mechanical and an Electrical Device



Graph comparing P_f of Mechanical and an Electrical Device



Comparing Failure Rates of an Electrical and a Mechanical Device

$$\text{Definition : Failure Rate } = \lambda(\text{t}) = \frac{\text{d/dt}\left(1 - \text{R(t)}\right)}{\text{R(t)}}$$

$$\text{Electrical Device - } \lambda(\text{t}) = \lambda$$

$$\text{Mechanical Device - } \lambda(\text{t}) = \frac{\dfrac{1}{s\sqrt{2\pi}}\,\text{e}^{-\left(\dfrac{(\text{t}-\text{u})^2}{2\,\text{s}^2}\right)}}{1 - \dfrac{1}{s\sqrt{2\pi}}\displaystyle\int_0^{\text{t}} \text{e}^{-\left(\dfrac{(\text{x}-\text{u})^2}{2\,\text{s}^2}\right)}\,\text{dx}}$$

Note: Mechanical device exhibiting a “Normal” failure characteristic

Calculating probability of failure of both Constant and Non-constant failure rate Devices

Calculating Failure Rate of Components

Calculating failure rates of components is performed in one of two ways.

Measurement:
For example place a number of identical components into operation, then measure and average their times to failure.

Prediction:
Most failure rate predictions are performed using a collection of formulas listed in Mil-Hdbk 217. These formulas were designed and developed based on the **physics** of failure of various components. Math modeling failure characteristics of components involves physics to a great extent.

Calculating Failure Rate of Components cont.

Example: Calculating Memory IC Failure Rate using Mil-Hdbk 217

$$\lambda_p = (C_1\pi_T + C_2\pi_E + \lambda_{CYC})\pi_Q\pi_L$$

$$C_1 = \text{Die Complexity Factor}$$

$$\pi_T = \text{Temperature Factor}$$

$$C_2 = \text{Package / \# Pins Factor}$$

$$\pi_E = \text{Environmental Factor}$$

$$\lambda_{CYC} = \left(A_1B_1 + \frac{A_2B_2}{\pi_Q} \right)\pi_{ECC} \;\; \text{(for EEProms only)}$$

$$\pi_Q = \text{Quality Factor (Procured according to what Standard)}$$

$$\pi_L = \text{Learning Factor (Years device in production)}$$

$$\pi_{ECC} = \text{Error Correction Code Factor}$$

$$A1 = 6.817 \text{ x } C \qquad C = \text{\# of programming cycles}$$

$$A2 = 0 \text{ if } C <= 300k$$

$$A2 = 1.1 \text{ if } C\ 300k < C <= 400$$

$$A2 = 2.3 \text{ if } C\ 400k < C <= 500k$$

Calculating Memory IC Failure Rate using Mil-Hdbk 217

B_1 and B_2 Factors for λ_{cyc} Calculation

	Flotox[1] (B_1)					Textured-Poly[2] (B_1)					Textured-Poly[3] (B_2)				
Memory Size, B(Bits) → T_J (°C)	4K	16K	64K	256K	1M	4K	16K	64K	256K	1M	4K	16K	64K	256K	1M
25	.27	0.55	1.1	2.2	4.3	.47	.66	.94	1.3	1.9	.54	0.76	1.1	1.5	2.1
30	.30	0.60	1.2	2.4	4.8	.50	.71	1.0	1.4	2.0	.50	0.71	1.0	1.4	2.0
35	.33	0.66	1.3	2.7	5.2	.54	.77	1.1	1.5	2.2	.47	0.67	.95	1.3	1.9
40	.36	0.72	1.4	2.9	5.7	.58	.82	1.2	1.6	2.3	.45	0.63	.89	1.3	1.8
45	.40	0.79	1.6	3.2	6.3	.62	.88	1.3	1.8	2.5	.42	0.59	.84	1.2	1.7
50	.43	0.86	1.7	3.4	6.8	.67	.95	1.3	1.9	2.7	.40	0.56	.80	1.1	1.6
55	.47	0.93	1.9	3.7	7.4	.71	1.0	1.4	2.0	2.8	.38	0.53	.75	1.1	1.5
60	.51	1.0	2.0	4.1	8.0	.76	1.1	1.5	2.1	3.0	.36	0.50	.72	1.0	1.4
65	.55	1.1	2.2	4.4	8.6	.81	1.1	1.6	2.3	3.2	.34	0.48	.68	.96	1.3
70	.59	1.2	2.4	4.7	9.3	.86	1.2	1.7	2.4	3.4	.32	0.45	.65	.91	1.3
75	.63	1.3	2.5	5.1	10	.91	1.3	1.8	2.6	3.6	.31	0.43	.62	.87	1.2
80	.68	1.4	2.7	5.4	11	.96	1.4	1.9	2.7	3.8	.29	0.41	.59	.83	1.2
85	.73	1.5	2.9	5.8	12	1.0	1.4	2.0	2.9	4.0	.28	0.39	.56	.79	1.1
90	.78	1.6	3.1	6.2	12	1.1	1.5	2.2	3.0	4.3	.27	0.38	.54	.75	1.1
95	.83	1.7	3.3	6.7	13	1.1	1.6	2.3	3.2	4.5	.26	0.36	.51	.72	1.0
100	.89	1.8	3.5	7.1	14	1.2	1.7	2.4	3.4	4.7	.25	0.35	.49	.69	.98
105	.94	1.9	3.8	7.5	15	1.3	1.8	2.5	3.5	5.0	.24	0.33	.47	.66	.94
110	1.0	2.0	4.0	8.0	16	1.3	1.9	2.6	3.7	5.2	.23	0.32	.45	.64	.90
115	1.1	2.1	4.2	8.5	17	1.4	1.9	2.8	3.9	5.5	.22	0.31	.44	.61	.86
120	1.1	2.2	4.5	9.0	18	1.4	2.0	2.9	4.1	5.7	.21	0.30	.42	.59	.83
125	1.2	2.4	4.7	9.5	19	1.5	2.1	3.0	4.3	6.0	.20	0.29	.41	.57	.80
130	1.3	2.5	5.0	10	20	1.6	2.2	3.2	4.4	6.3	.19	0.27	.39	.55	.77
135	1.3	2.6	5.3	11	21	1.6	2.3	3.3	4.6	6.5	.19	0.27	.38	.53	.75
140	1.4	2.8	5.6	11	22	1.7	2.4	3.4	4.8	6.8	.18	0.26	.36	.51	.72
145	1.5	2.9	5.8	12	23	1.8	2.5	3.6	5.0	7.1	.18	0.25	.35	.50	.70
150	1.5	3.1	6.1	12	24	1.9	2.6	3.7	5.2	7.4	.17	0.24	.34	.48	.68
155	1.6	3.2	6.4	13	26	1.9	2.7	3.9	5.4	7.7	.16	0.23	.33	.46	.65
160	1.7	3.4	6.8	14	27	2.0	2.8	4.0	5.6	8.0	.16	0.23	.32	.45	.63
165	1.8	3.5	7.1	14	28	2.1	2.9	4.2	5.9	8.2	.15	0.22	.31	.44	.61
170	1.9	3.7	7.4	15	29	2.2	3.0	4.3	6.1	8.6	.15	0.21	.30	.42	.60
175	1.9	3.9	7.7	15	31	2.2	3.1	4.5	6.3	8.9	.15	0.21	.29	.41	.58

1. $$B_1 = \left(\frac{B}{16000}\right)^{.5}\left[\exp\left(\frac{-.15}{8.617 \times 10^{-5}}\left(\frac{1}{T_J + 273} - \frac{1}{333}\right)\right)\right]$$

2. $$B_1 = \left(\frac{B}{64000}\right)^{.25}\left[\exp\left(\frac{-.12}{8.617 \times 10^{-5}}\left(\frac{1}{T_J + 273} - \frac{1}{303}\right)\right)\right]$$

3. $$B_2 = \left(\frac{B}{64000}\right)^{.25}\left[\exp\left(\frac{.1}{8.617 \times 10^{-5}}\left(\frac{1}{T_J + 273} - \frac{1}{303}\right)\right)\right]$$

T_J = Worse Case Junction Temperature (°C). See Section 5.11 for T_J Determination

B = Number of bits. NOTE: 1K = 1024 bits

Constant Failure Rate Devices (Exponential Distribution)

The failure characteristics of many electrical components follows very closely the exponential curve, and therefore the calculation of probability of success P_s and probability of failure P_f is very simple:

$$\text{Rel} = P_s = e^{-\lambda t}$$

$$\text{UnRel} = P_f = 1 - e^{-\lambda t}$$

Unfortunately this is not the case with mechanical components i.e. non-constant failure rate devices.

Non-constant Failure Rate Devices

Excerpt from Mil-Hdbk 217:

The following failure-rate model applies to **motors** with power ratings below one horsepower. The model is dictated by two failure modes, bearing failures and winding failures. Typical applications include fans and blowers as well as various other motor applications.

The instantaneous failure rates, or hazard rates, experienced by motors are not constant but increase with time.

$$\lambda_p = \left[\frac{t^2}{\alpha_B{}^3} + \frac{1}{\alpha_W} \right] x\, 10^6 \text{ Failures}/10^6 \text{ Hours}$$

α_B = Bearing Factor

α_W = Winding Factor

Non-constant Failure Rate Devices

- The failure characteristics of many mechanical components follows very closely to the Normal curve. A graph representing the number of failures vs. time will result in the famous bell curve which we call the Normal distribution.
- The Normal equation is

$$\mathrm{f}(x) = \frac{1}{s\sqrt{2\pi}}\, e^{-\left[\frac{(x-u)^2}{2s^2}\right]}$$

u= Mean of Distribution
s= Standard Deviation

- The Probability of Failure (P_f) is the Integral from 0 to t of the above equation.

$$P_f = \frac{1}{s\sqrt{2\pi}} \int_0^t e^{-\left[\frac{(x-u)^2}{2s^2}\right]}\, dx$$

Application: Failure time distribution of items whose failure modes are a result of wearout

Normal Distribution Example

- An item has a mean wearout life of 300 hours with a standard deviation of 40 hours. If the time before its maintenance scheduled replacement is 200 hours, the probability it will meet its maintenance time is:

(no failures in 200 hours)

$$R(200) = 1 - F(200) = 1 - \frac{1}{40\sqrt{2\pi}} \int_0^{200} e^{-\left[\frac{(x-300)^2}{2 \bullet 40^2}\right]} dx$$

$$\text{Ans} : 0.99379$$

(no failures in 250 hours)

$$R(250) = 1 - F(250) = 1 - \frac{1}{40\sqrt{2\pi}} \int_0^{250} e^{-\left[\frac{(x-300)^2}{2 \bullet 40^2}\right]} dx$$

$$\text{Ans} : 0.89435$$

Two Major Problems with MTBF Predictions

Problem 1 with MTBF Predictions

Quote from Wikipedia:

As of 1995, the use of MTBF in the aeronautical industry (and others) has been called into question due to the inaccuracy of its application to real systems and the nature of the culture which it engenders. Many component MTBFs are given in databases, and often these values are very inaccurate.

This has led to the negative exponential distribution being used much more than it should have been. Some estimates say that only 40% of components have failure rates described by this. It has also been corrupted into the notion of an "acceptable" level of failures, which removes the desire to get to the root cause of a problem and take measures to erase it. The British Royal Air Force is looking at other methods to describe reliability, such as maintenance-free operating period (MFOP).

Problem 1 restated more simply:
Some non-constant failure rate devices have been and still are erroneously modeled as constant failure rate devices.

Problem 1 with MTBF Predictions cont.

Quote: (ASQ Reliability Review, Vol. 24, No. 1, pp 18-23, March 2004)

Many years ago people hypothesized the constant failure rate model for electronics parts made to military standards observed to have constant or bathtub-shaped failure rates. Diligent people collected data and used statistics to estimate constant failure rates and used regression to estimate the p-factors and stress factors according to acceleration models.

MIL-HDBK-217 standardized MTBF prediction, under the assumptions of series systems of statistically independent parts and constant failure rates. <u>Unfortunately, many parts don't have constant failure rates.</u> Some have infant mortality. Some deteriorate; such as motors (dirt, lubricants, and bearings), some capacitors (electrolytic), and ICs (electromigration and other physical and chemical processes).

Problem 2 with MTBF Predictions

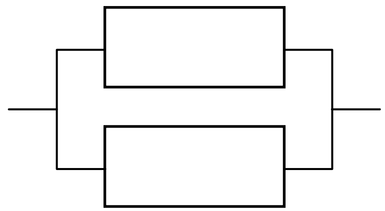
Quote: Quanterion Solutions Inc. Vol.1 No. 1, August 2001

MIL-HDBK-217 has been the mainstay of reliability predictions for about 40 years but it has not been updated since 1995, and there are no plans by the military to update it in the future.

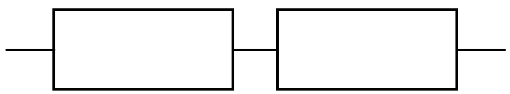
Problem 2: MIL-HDBK-217 is clearly out of date. Evidence has shown that 217 predicted data can differ from field data by as much as 10 times.

Intro to Non-combinatorial Probability and its application to Reliability

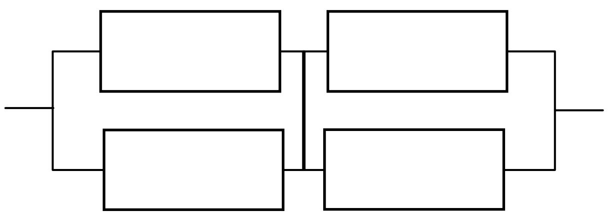
Combinatorial vs. Non-combinatorial Logic



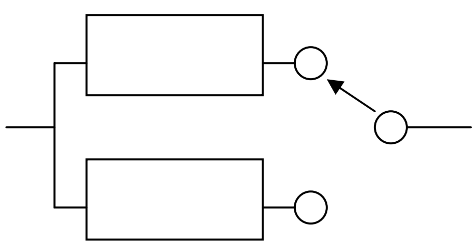
Parallel Combinatorial



Series Combinatorial



Series / Parallel Combinatorial



Standby Non-combinatorial

Note: Distinction required for proper system math modeling

Combinatorial vs. Non-combinatorial Logic

Combinatorial Logic

Two or more input states define one or more output states.
Output states are related by defined rules that are independent of previous states.

- Logic depends solely on combinations of inputs
- Time is neither modeled or recognized
- Outputs change when inputs change irrespective of time
- Output is a function of, and only of, the present input

Simply stated combinatorial logic is a logic that can be expressed with any combination of And gates and Or gates.

Non-combinatorial Logic (Sequential Logic)

Logic output(s) depends on combinations of present input states, <u>and</u> combinations of previous input states. In other words non-combinatorial logic has memory while combinatorial logic does not.

Non-combinatorial logic cannot be expressed <u>exactly</u> with any combination of And gates and Or gates.

Combinatorial vs. Non-combinatorial logic

(Combinatorial Example 1)

Three (3) identical black boxes are operating "Active Redundant". What is the probability that at least one black box will operate if the reliability (probability of success) of one box is 0.9?

- Probability of success = p = 0.9
- Probability of failure = q = 1 – p = 0.1

$1 = (p + q)^3$

$1 = 1p^3 + 3p^2q + 3pq^2 + 1q^3$

$1 = 1(.9)^3 + 3(.9)^2(.1) + 3(.9)(.1)^2 + 1(.1)^3$

P (0 failures)

P (1 failure)

P (2 failures)

P (3 failures)

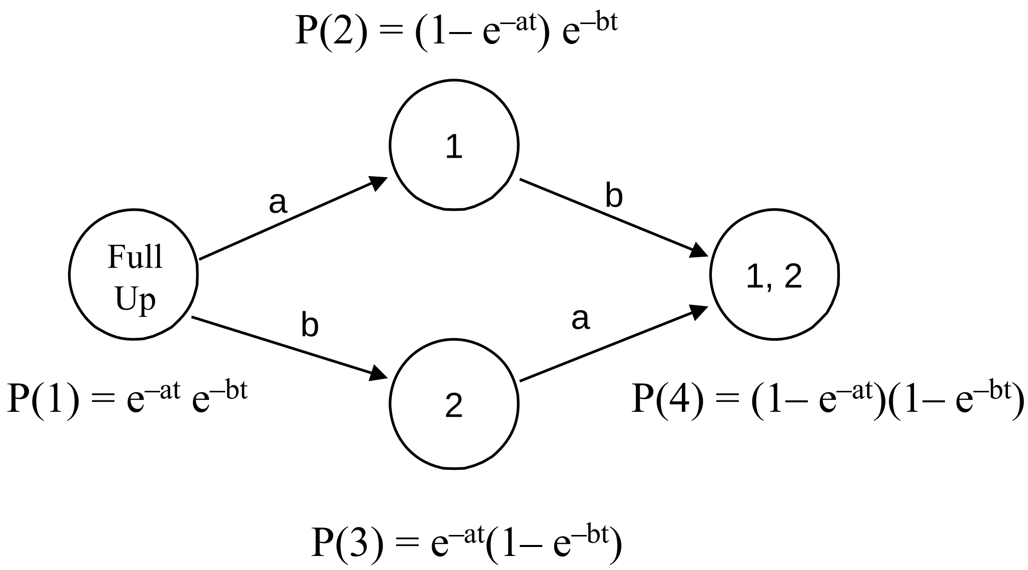
Note: This is a combinatorial calculation and can only be used when all failure rates are the same and not subject to changes.

Combinatorial vs. Non-combinatorial logic

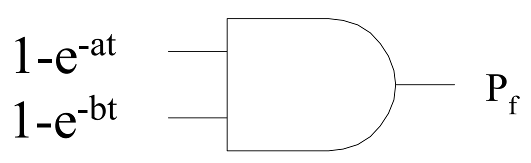
(Combinatorial Example 2)

Two black boxes start operation at the same time. Box 1 has failure rate a and Box 2 has failure rate b. Successful system operation requires that Box 1 or Box 2 or both be working. Find P_f the Probability of System Failure.

State Diagram



Logic



$$P_f = (1- e^{-at})(1- e^{-bt})$$

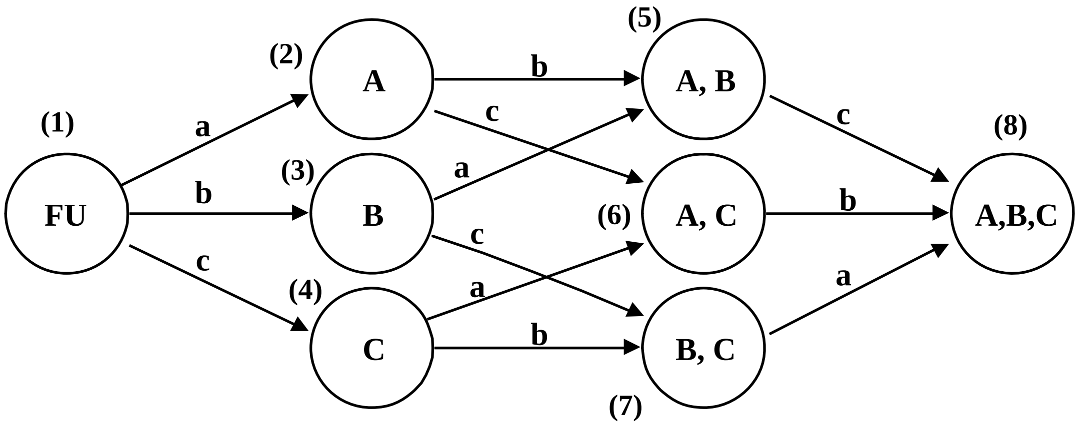
Note: Failure rates remain unchanged regardless of state.

Combinatorial vs. Non-combinatorial logic

(Combinatorial Example 3)

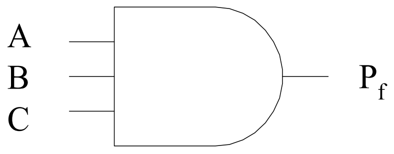
Three black boxes start operation at the same time. Box A, B, and C have failure rate a, b, and c respectively. Successful system operation requires that Box A, B, or C be working. Find P_f the Probability of System Failure.

State Diagram



$$P_f = P(8) = (1-e^{-at})(1-e^{-bt})(1-e^{-ct})$$

Logic



$$A = (1-e^{-at})$$
$$B = (1-e^{-bt})$$
$$C = (1-e^{-ct})$$
$$P_f = P(8)$$

Note: Failure rates remain unchanged regardless of state.

Combinatorial vs. Non-combinatorial logic

(Combinatorial Example 3 cont.)

- State probabilities for components in paralleled are:
  - State 1 – No failures P(1) = $e^{-at} \cdot e^{-bt} \cdot e^{-ct}$
  - State 2 – Box A fails (P2) = $(1-e^{-at}) \cdot e^{-bt} \cdot e^{-ct}$
  - State 3 – Box B fails (P3) = $(1-e^{-bt}) \cdot e^{-at} \cdot e^{-ct}$
  - State 4 – Box C fails (P4) = $(1-e^{-ct}) \cdot e^{-at} \cdot e^{-bt}$
  - State 5 – Boxes A & B fail (P5) = $(1-e^{-at})(1-e^{-bt}) \cdot e^{-ct}$
  - State 6 – Boxes A & C fail (P6) = $(1-e^{-at})(1-e^{-ct}) \cdot e^{-bt}$
  - State 7 – Boxes B & C fail (P7) = $(1-e^{-bt})(1-e^{-ct}) \cdot e^{-at}$
  - State 8 – All 3 Boxes failed (P8) = $(1-e^{-at})(1-e^{-bt})(1-e^{-ct})$

Combinatorial vs. Non-combinatorial logic

(Combinatorial Example 3 cont.)

With respect to the previous slide assume all failure rates are the same i.e. a = b = c, and let $p = e^{-at}$ and $q = 1-e^{-at}$ then

- State 1 — 0 failures $= e^{-at} \cdot e^{-at} \cdot e^{-at} = (e^{-at})^3 = p^3$
- State 2+3+4 — 1 failures $= 3(e^{-at})^2(1-e^{-at}) = 3p^2q$
- State 5+6+7 — 2 failures $= 3(e^{-at})(1-e^{-at})^2 = 3pq^2$
- State 8 — 3 failures $= (1-e^{-at})^3 = q^3$

Note:

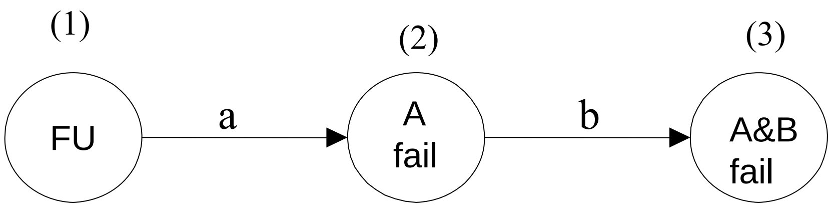
Probability distributions become the same as when using a binomial expansion

Combinatorial vs. Non-combinatorial logic

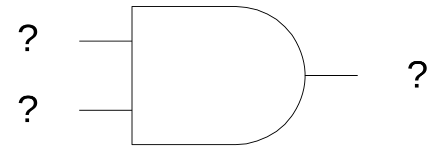
(Non - combinatorial Example 1)

Box A has failure rate a and Box B has failure rate b. Box A is turned on while Box B remains powered off in standby mode. Immediately upon detection of Box A failure, Box B is turned on. Calculate the probability that both boxes are failed.

State Diagram



Logic



$$P(1) = e^{-at}$$

$$P(2) = \frac{a}{a-b}\left(e^{-bt} - e^{-at}\right)$$

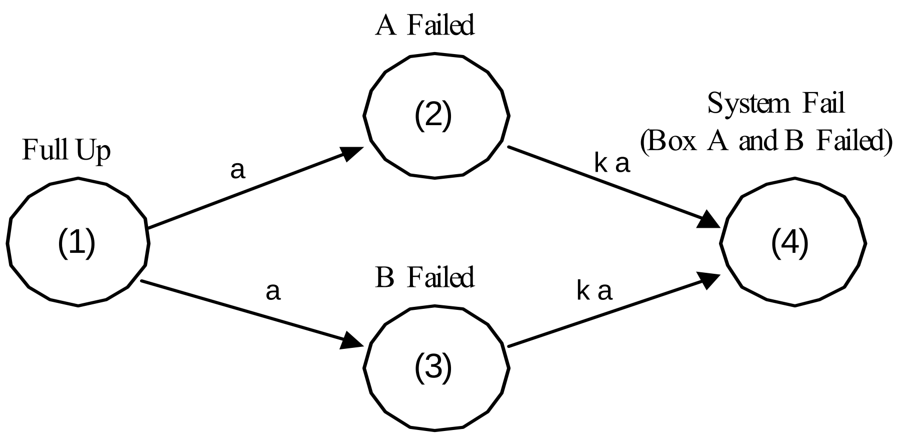
$$P(3) = \frac{b}{a-b}e^{-at} - \frac{a}{a-b}e^{-bt}$$

Combinatorial vs. Non-combinatorial logic

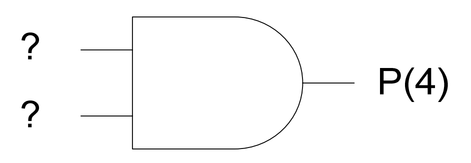
(Non - combinatorial Example 2)

Consider a parallel load-sharing system consisting of two components A and B. Under the load sharing conditions, each component has failure rate "a". Upon failure of one component, the failure rate of the surviving component is "ka" (k times a) due to increased stress.

State Diagram



Logic



$$P(4) = 1 + \frac{k}{2-k} e^{-2at} - \frac{2}{2-k} e^{-kat}$$

$$P_f = P(4) = ?$$

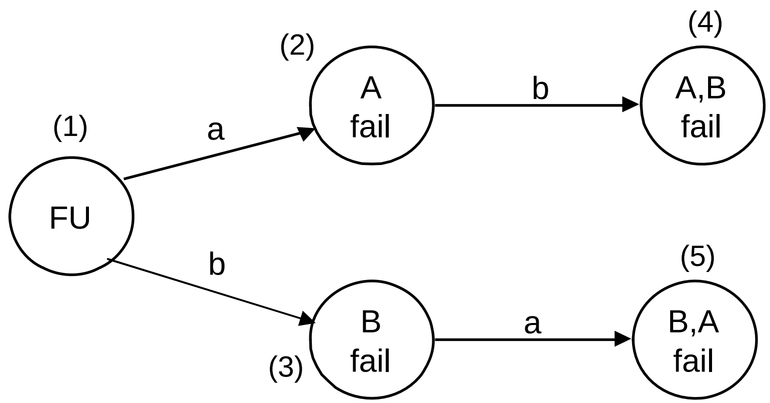
Note: k = 1 implies no change in failure rate. In that case, this problem becomes combinatorial and can be solved directly using the binomial expansion.

Combinatorial vs. Non-combinatorial logic

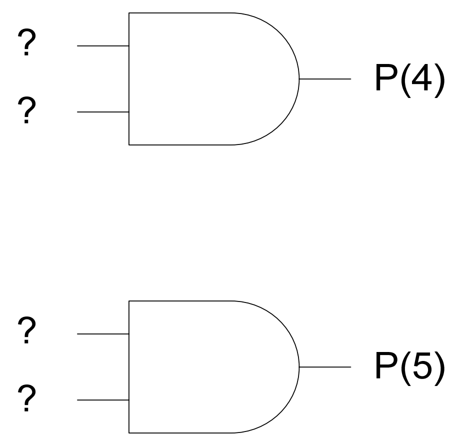
(Non - combinatorial Example 3)

Two components are in operation. Find the probability that both Boxes A and B fail <u>and</u> that Box A fails before Box B. Also find the probability that both Boxes fail <u>and</u> that Box B fails before Box A.

State Diagram



Logic



$$P(4) = \frac{a}{a+b} + \frac{b}{a+b} e^{-(a+b)t} - e^{-bt}$$

$$P(5) = \frac{b}{a+b} + \frac{a}{a+b} e^{-(a+b)t} - e^{-at}$$

Why Markov Analysis?

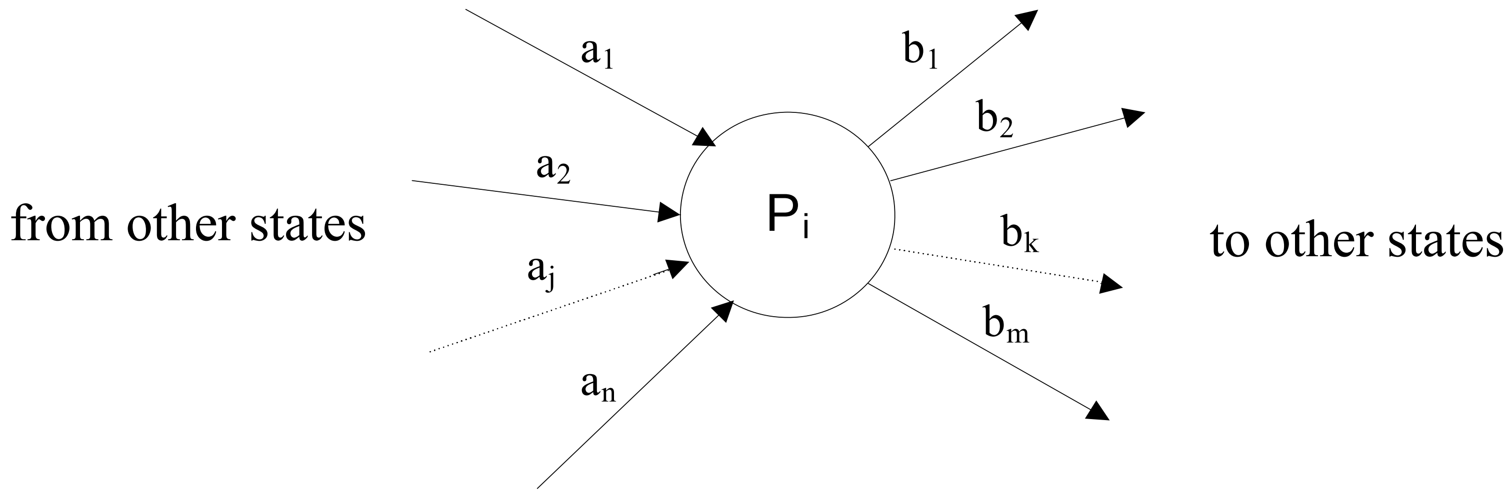
for

Calculating Probability of

Non-combinatorial Problems

Solving a Non-combinatorial Problem using DEs

The following is a typical Markov State taken from a Markov State Diagram with n input transitions with constant failure rates a_j, and m output transitions with constant failure rates b_k.



Pi (the probability of being in state i) cannot be calculated immediately. Calculation of Pi requires the solution of a set of simultaneous differential equations (DE). Determination of the DEs is a very simple procedure once the Markov State Diagram of a system has been constructed.

Solving Standby Problem using DEs

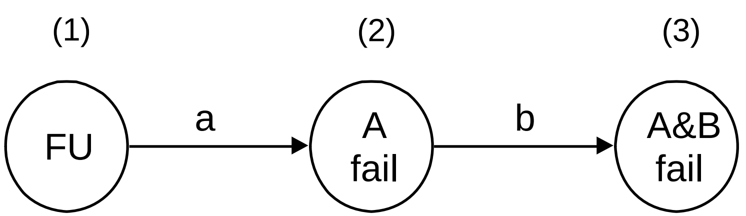
There is a one to one correspondence between each Markov state of a system and its associated DE. The DE associated with a typical P(i) is:

$$\frac{dP(i)}{dt} = \sum_{j=1}^{n} a_j P(j) - \left(\sum_{k=1}^{m} b_k \right) P(i)$$

For the sake of simplifying notation let $P_i = P(i)$.

Therefore with respect to the **Standby Problem**, the system's set of DEs are easily determined from its Markov State Diagram.

State Diagram



DEs

$$\frac{dP_1}{dt} = -aP_1, \quad \frac{dP_2}{dt} = aP_1 - bP_2, \quad \frac{dP_3}{dt} = bP_3$$

Note: Transitions <u>into</u> a state result in <u>positive</u> terms in the DE, while transitions <u>leaving</u> a state yield <u>negative</u> terms.

Solving Simultaneous DEs using Matrix Algebra

What follows is a method using Matrix algebra for solving for P_1, P_2, and P_3 numerically based on the 3 Simultaneous DEs obtained from the Markov Diagram:

$$\frac{dP_1}{dt} = -aP_1, \quad \frac{dP_2}{dt} = aP_1 - bP_2, \quad \frac{dP_3}{dt} = bP_2 \quad \text{therefore} \quad P'(t) = A \cdot P(t)$$

$$\text{where} \quad P'(t) = \begin{vmatrix} \frac{dP_1}{dt} \\ \frac{dP_2}{dt} \\ \frac{dP_3}{dt} \end{vmatrix} \quad A = \begin{vmatrix} -a, & 0, & 0 \\ a, & -b, & 0 \\ 0, & b, & 0 \end{vmatrix} \quad P(t) = \begin{vmatrix} P_1 \\ P_2 \\ P_3 \end{vmatrix} \quad \text{and the solution is}$$

$$P(t) = \left| \exp(At) \right| \cdot P(0) \quad \text{where} \quad \exp(At) = I + At + \frac{A^2 t^2}{2!} + \frac{A^3 t^3}{3!} + \cdots$$

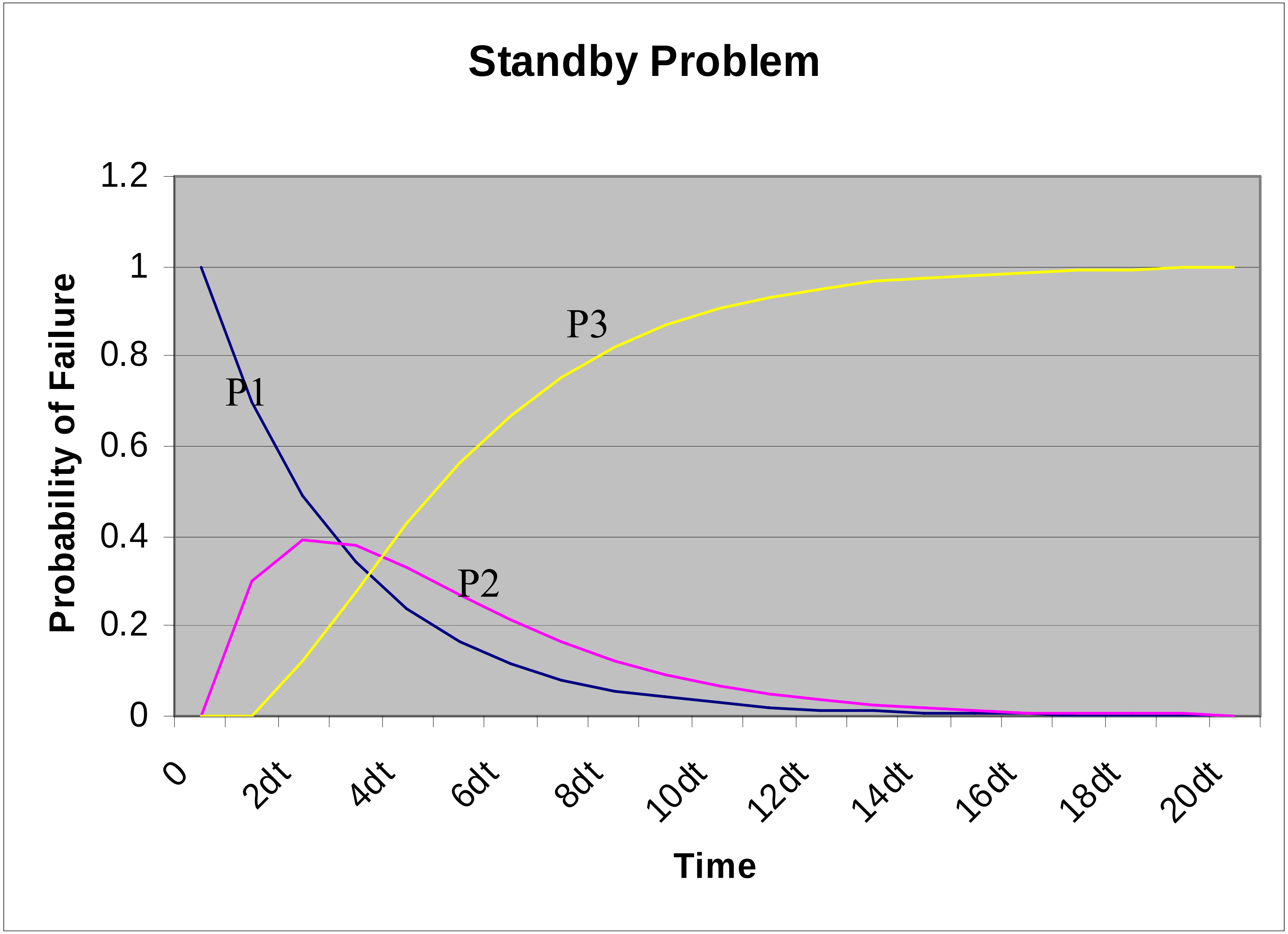
Note: $P_1(0) = 1$, $P_2(0) = 0$, and $P_3(0) = 0$ assumed.

Solving Simultaneous DEs using Arithmetic

$$dP_1 = -aP_1dt, \quad dP_2 = (aP_1 - bP_2)dt, \quad dP_3 = bP_2dt \qquad dt = 1,\ a = 0.3,\ b= 0.4$$

	P1-a*P1*dt	P2+(a*P1-b*P2)*dt	P3+b*P2*dt	Check
t	P1	P2	P3	P1+P2+P3
0	1	0	0	1
dt	0.7	0.3	0	1
2dt	0.49	0.39	0.12	1
3dt	0.343	0.381	0.276	1
4dt	0.2401	0.3315	0.4284	1
5dt	0.16807	0.27093	0.561	1
6dt	0.117649	0.212979	0.669372	1
7dt	0.0823543	0.1630821	0.7545636	1
8dt	0.05764801	0.12255555	0.81979644	1
9dt	0.040353607	0.090827733	0.86881866	1
10dt	0.028247525	0.066602722	0.905149753	1
11dt	0.019773267	0.048435891	0.931790842	1
12dt	0.013841287	0.034993515	0.951165198	1
13dt	0.009688901	0.025148495	0.965162604	1
14dt	0.006782231	0.017995767	0.975222002	1
15dt	0.004747562	0.01283213	0.982420309	1
16dt	0.003323293	0.009123546	0.987553161	1
17dt	0.002326305	0.006471116	0.991202579	1
18dt	0.001628414	0.004580561	0.993791025	1
19dt	0.00113989	0.003236861	0.99562325	1
20dt	0.000797923	0.002284083	0.996917994	1

Solving Simultaneous DEs using Arithmetic cont.



Solving Simultaneous DEs using Laplace

What follows is a method using Laplace Transforms for solving for P_1, P_2, and P_3 based on the 3 Simultaneous DEs obtained from the Markov Diagram:

$$\frac{dP_1}{dt} = -aP_1, \quad \frac{dP_2}{dt} = aP_1 - bP_2, \quad \frac{dP_3}{dt} = bP_2 \Rightarrow$$

$$L\left(\frac{dP_1}{dt}\right) = L(-aP_1) \Rightarrow sL(P_1) - P_1(0) = -aL(P_1) \Rightarrow sL(P_1) - 1 = -aL(P_1) \qquad (1)$$

$$L\left(\frac{dP_2}{dt}\right) = L(aP_1 - bP_2) \Rightarrow sL(P_2) - P_2(0) = aL(P_1) - bL(P_2) \Rightarrow$$

$$sL(P_2) = aL(P_1) - bL(P_2) \qquad (2)$$

$$L\left(\frac{dP_3}{dt}\right) = L(bP_2) \Rightarrow sL(P_3) - P_3(0) = bL(P_2) \Rightarrow sL(P_3) = bL(P_2) \qquad (3)$$

$$(1) \Rightarrow L(P_1) = \frac{1}{s+a} \quad \text{and } (1) \,\&\, (2) \Rightarrow L(P_2) = \frac{a}{(s+a)(s+b)} \qquad (4)$$

Note: $P_1(0) = 1$, $P_2(0) = 0$, and $P_3(0) = 0$ assumed.

Solving Simultaneous DEs using Laplace cont.

$$(4) \Rightarrow P_1 = L^{-1}\left(\frac{1}{s+a}\right) = e^{-at} \text{ and } P_2 = L^{-1}\left(\frac{a}{(s+a)(s+b)}\right)$$

Using techniques from Partial Fractions $\dfrac{a}{(s+a)(s+b)} = \dfrac{a/(a-b)}{s+b} - \dfrac{a/(a-b)}{s+a} \Rightarrow$

$$P_2 = L^{-1}\left(\frac{a/(a-b)}{s+b}\right) - L^{-1}\left(\frac{a/(a-b)}{s+a}\right) \Rightarrow P_2 = \frac{a}{a-b}e^{-bt} - \frac{a}{a-b}e^{-at}$$

Note: The third DE in Line (3) could be used to solve for P3. However since P_1 and P_2 are known, use the fact that $P_1 + P_2 + P_3 = 1$. This approach is faster and simpler.

$$P_1 + P_2 + P_3 = 1 \Rightarrow P_3 = 1 - e^{-at} + \frac{a}{a-b}e^{-at} - \frac{a}{a-b}e^{-bt} \Rightarrow$$

$$\boxed{P_3 = 1 + \frac{b}{a-b}e^{-at} - \frac{a}{a-b}e^{-bt}}$$

Solution to "Standby" Using Formula

Many Markov problems can be solved using the following formula:

If f(t) and g(t) are functions of t, and $\frac{dPi}{dt} = g(t) - f(t)\ P_i$

then $P_i\ e^{\int f(t)dt} = \int g(t)\ e^{\int f(t)dt}\ dt + C$ C = arbitrary constant

$$\frac{dP_1}{dt} = -aP_1 \Rightarrow g(t) = 0 \text{ and } f(t) = a \Rightarrow P_1\ e^{\int a dt} = C_1 \Rightarrow P_1\ e^{at} = C_1 \Rightarrow P_1 = C_1 e^{-at}$$

Where C_1 = probability of P_1 at $t = 0$ Assume $C_1 = P_1(0) = 1 \Rightarrow P_1 = e^{-at}$

$$\frac{dP_2}{dt} = aP_1 - bP_2 \Rightarrow g(t) = aP_1 \text{ and } f(t) = b \Rightarrow P_2\ e^{bt} = \int aP_1 e^{bt}\ dt + C_2$$

$$= a\int e^{-at} e^{bt}\ dt + C_2 = a\int e^{(b-a)t}\ dt + C_2 = \frac{a}{b-a} e^{(b-a)t} + C_2 \Rightarrow$$

Solution to "Standby" Using Formula cont.

$$P_2 = \frac{a}{b-a}e^{-at} + C_2 e^{-bt} \qquad \text{Now by assumption } P_2 = 0 \text{ when } t = 0 \Rightarrow C_2 = \frac{-a}{b-a} \Rightarrow$$

$$P_2 = \frac{a}{b-a}e^{-at} - \frac{a}{b-a}e^{-bt} = \frac{a}{a-b}e^{-bt} - \frac{a}{a-b}e^{-at}$$

Again since P_1 and P_2 are known, use the fact that $P_1 + P_2 + P_3 = 1$.

$$P_1 + P_2 + P_3 = 1 \Rightarrow P_3 = 1 - e^{-at} + \frac{a}{a-b}e^{-at} - \frac{a}{a-b}e^{-bt} \Rightarrow$$

$$\boxed{P_3 = 1 + \frac{b}{a-b}e^{-at} - \frac{a}{a-b}e^{-bt}}$$

Solution to "Standby" Using Convolution

A process called "Convolution" can also be used to calculate P_f of Standby Systems.

Definition:

Let A(t) and B(t) be probabilities of failure of two devices, with device B in Standby of device A, and let a(t) be the derivative of A(t).

The Convolution of A and B = Conv(t) = $\int_0^t B(t-x)\cdot a(x)dx = P_f$

Conv(t) turns out to be the Standby System's Probability of failure P_f.

Note: Convolution will be explained in more detail in the discussion of non-constant failure rate devices.

Solution to Standby Using Convolution cont.

Let $A(x) = 1–e^{–ax}$, and $B(x) = 1–e^{–bx}$ be the probabilities of failure of devices A and B. Then $A'(x) = a(x) = ae^{–ax}$, and $B(t–x) = 1–e^{–b(t–x)}$ since a and b are constant failure rates of devices A and B respectively $\Rightarrow$

$$P_f = \mathrm{Conv}(t) = \int_0^t (1 - e^{-b(t-x)}) \cdot ae^{-ax} dx = a\int_0^t (e^{-ax} - e^{-b(t-x)-ax})dx \Rightarrow$$

$$P_f = a\int_0^t (e^{-ax} - e^{-bt-(a-b)x})dx = a\int_0^t e^{-ax} dx - a \cdot e^{-bt} \int_0^t e^{-(a-b)x} dx \Rightarrow$$

$$P_f = (1 - e^{-at}) - \frac{a}{a-b} \cdot e^{-bt} (1 - e^{-(a-b)t}) = 1 - e^{-at} - \frac{a}{a-b}(e^{-bt} - e^{-at}) \Rightarrow$$

$$\boxed{P_f(sys) = 1 - \frac{a}{a-b} e^{-bt} + \frac{b}{a-b} e^{-at}}$$

Solution to Standby Using Convolution cont.

Another approach uses the famous "Convolution Theorem" that utilizes Laplace and Inverse Laplace Transforms. Simply stated:

$$\text{If } F(t) = \int_0^t B(t-x)\cdot a(x)dx \text{ then } F(t) = L^{-1}\{L[B(t)]\cdot L[a(t)]\}$$

$$\text{and } L^{-1}\{\, L[B(t)]\cdot L[a(t)] \,\} = L^{-1}\{\, L(1-e^{-bt})\cdot L(ae^{-at}) \,\} =$$

$$L^{-1}\left\{\left(\frac{1}{s}-\frac{1}{s+b}\right)\left(\frac{a}{s+a}\right)\right\} = L^{-1}\left\{\frac{a}{s(s+a)}-\frac{a}{(s+a)(s+b)}\right\} =$$

$$1-e^{-at}-\frac{a}{b-a}(e^{-at}-e^{-bt}) \Rightarrow \boxed{F(t) = 1+\frac{a}{b-a}e^{-bt}-\frac{b}{b-a}e^{-at}}$$

Probability Density Functions (PDF)

&

Cumulative Density Functions (CDF)

Probability Density Function (PDF)

Definition:

The mathematical definition of a continuous probability density function f(x), is a function that satisfies the following properties:

a) The probability that x is between two points a and b is less than or equal to 1.

b) f(x) is non-negative for all x.

c) The integral of the probability function is one, i.e. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Notes:

1) A probability density function is also known as a probability function.

2) The probability at a single point is always zero.

3) Probabilities are measured over intervals, not single points. That is, the area under the curve between two distinct points defines the probability for that interval.

Probability Density Function (PDF)

(Normal Distribution Example)

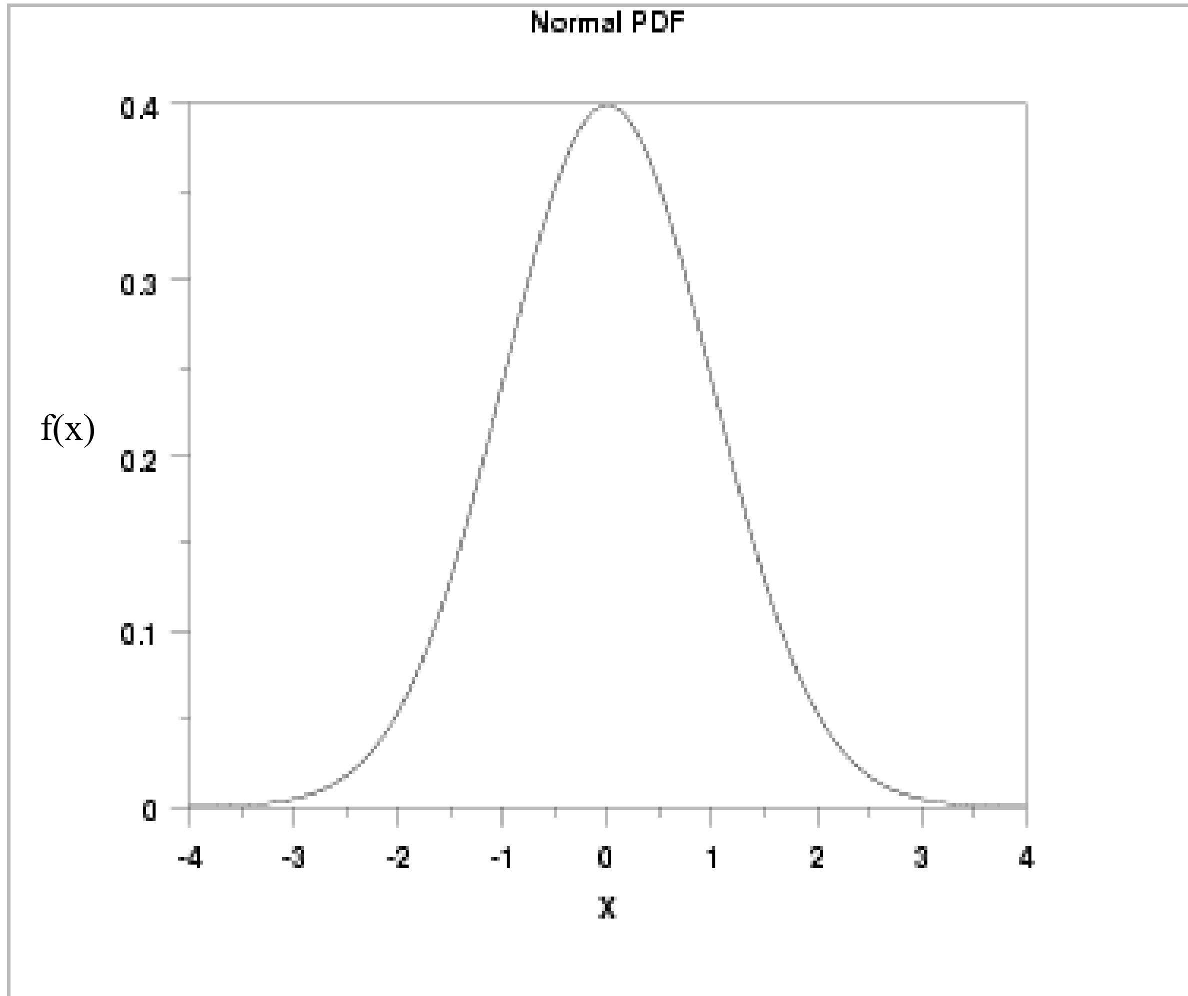
$$f(x)=\frac{1}{s\sqrt{2\pi}}e^{-\left(\frac{(x-u)^2}{2s^2}\right)}$$

u = mean

s = standard deviation

Note:

Area under curve = 1

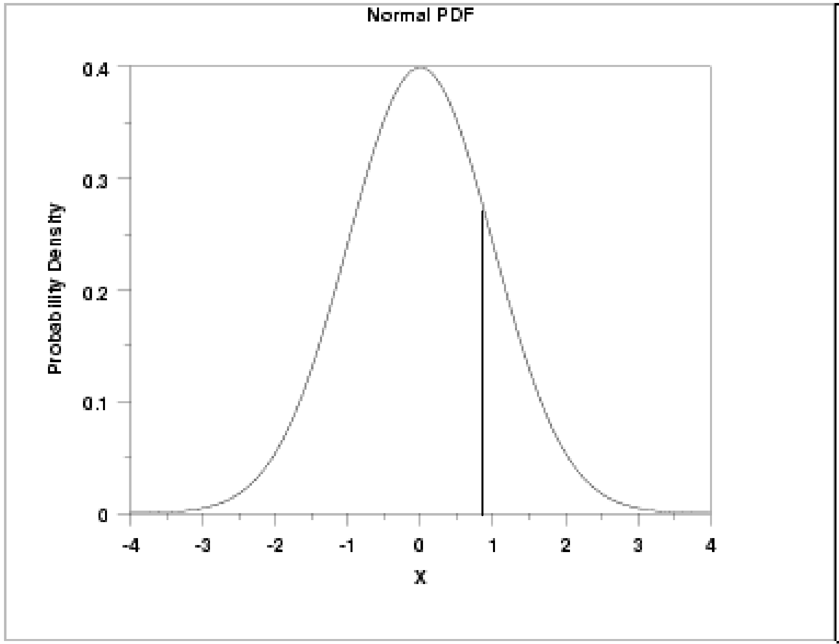


Cumulative Density Function (CDF)

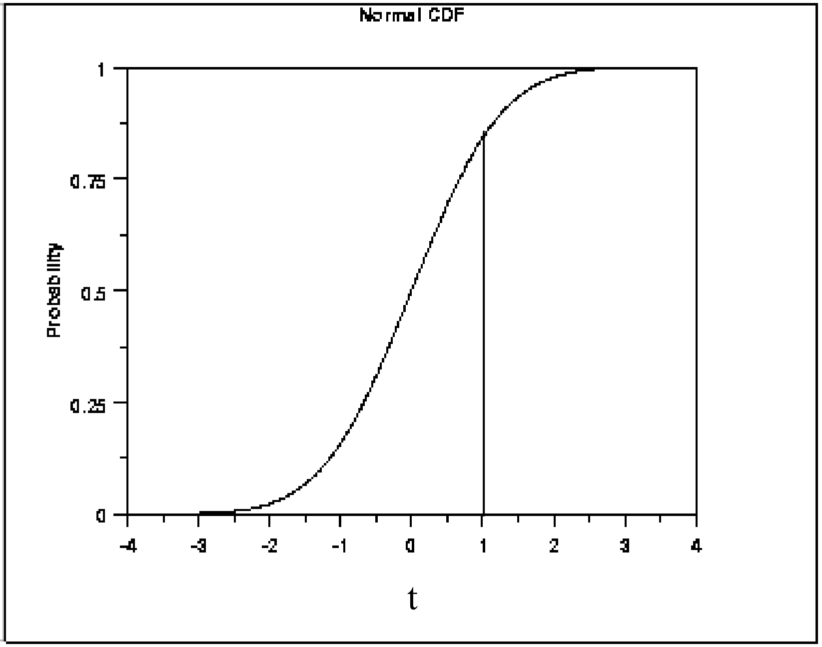
Definition:

The cumulative distribution function (CDF) is the probability that the variable takes a value less than or equal to t. For a continuous distribution, this can be expressed mathematically as $\mathrm{CDF} = \int_{-\infty}^{t} f(x)\, dx$

PDF



CDF

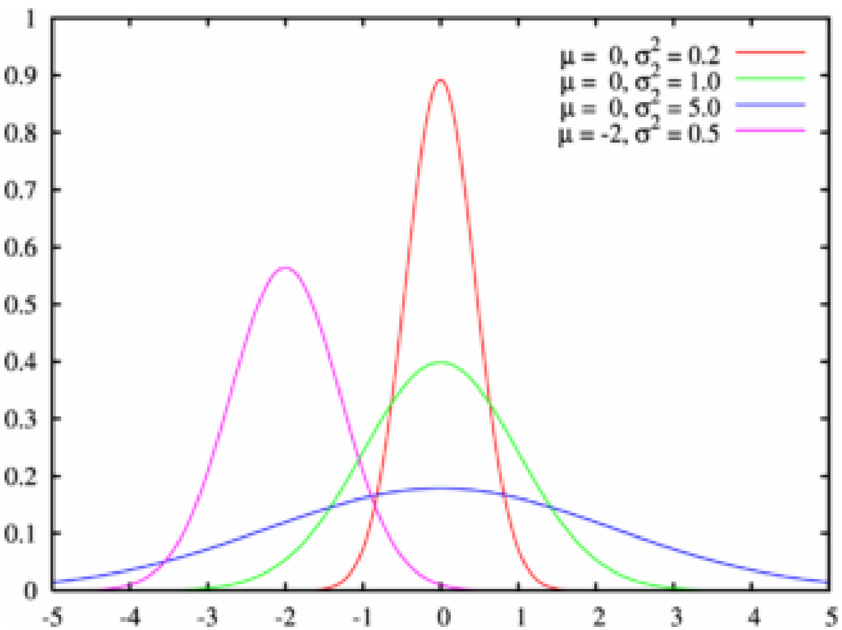


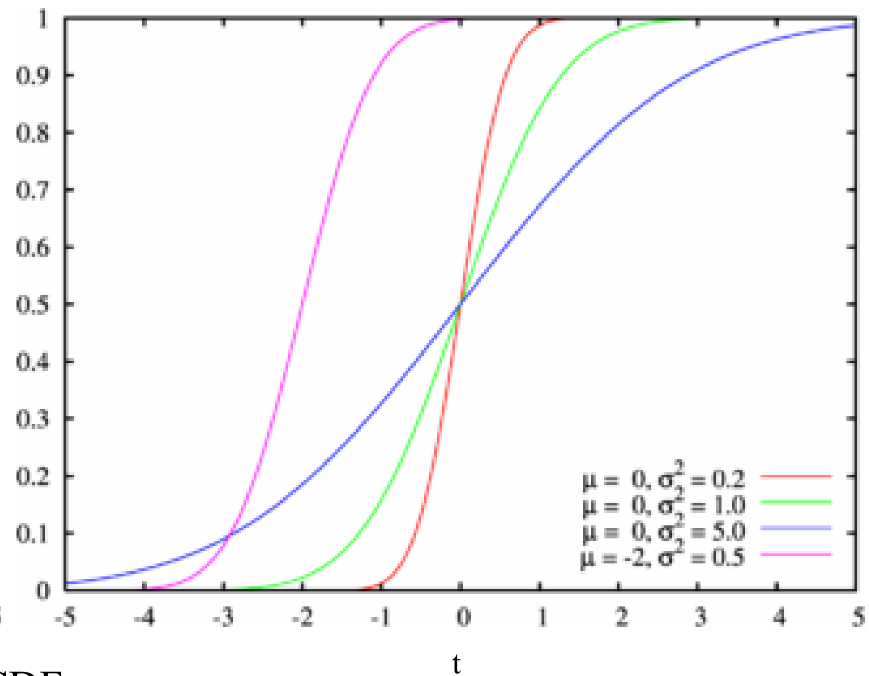
Cumulative Density Function (CDF)

More Normal Distribution Examples:

What is known as a CDF in the world of Probability is known as probability of failure P_f in the world of Reliability.

$$\text{PDF} = f(x) \qquad\qquad \text{CDF} = P_f = \int_{-\infty}^{t} f(x)\,dx$$

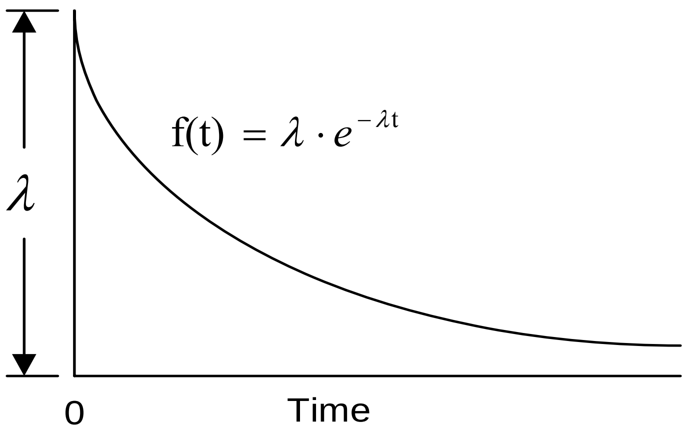




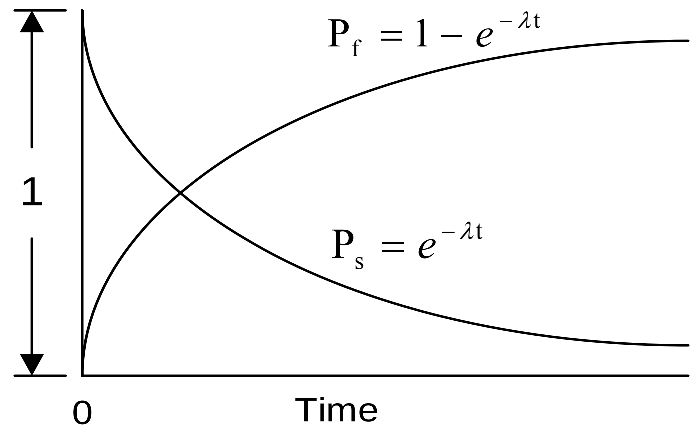
Note: The PDF is simply the derivative of the CDF

Exponential Distribution Example

Probability Density Function (PDF)



Probability of Failure Function (CDF)

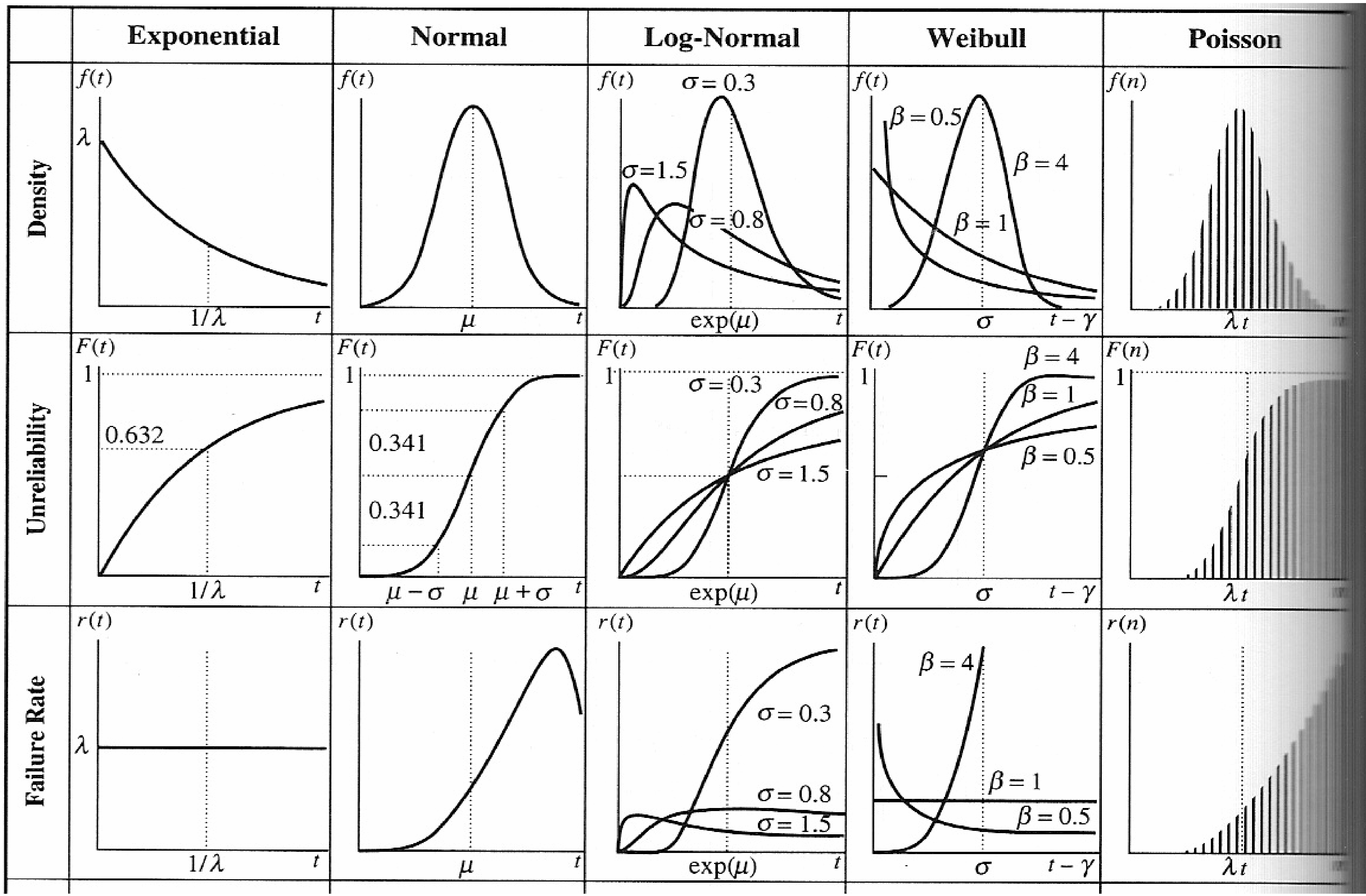


A graph of typical electrical/electronic components results in the Probability Density Function (PDF), whose curve is shown on the left. When one integrates the PDF over time, the result is a Continuous Distribution Function (CDF). The probability that a failure will occur at any time during the interval (0, t) is

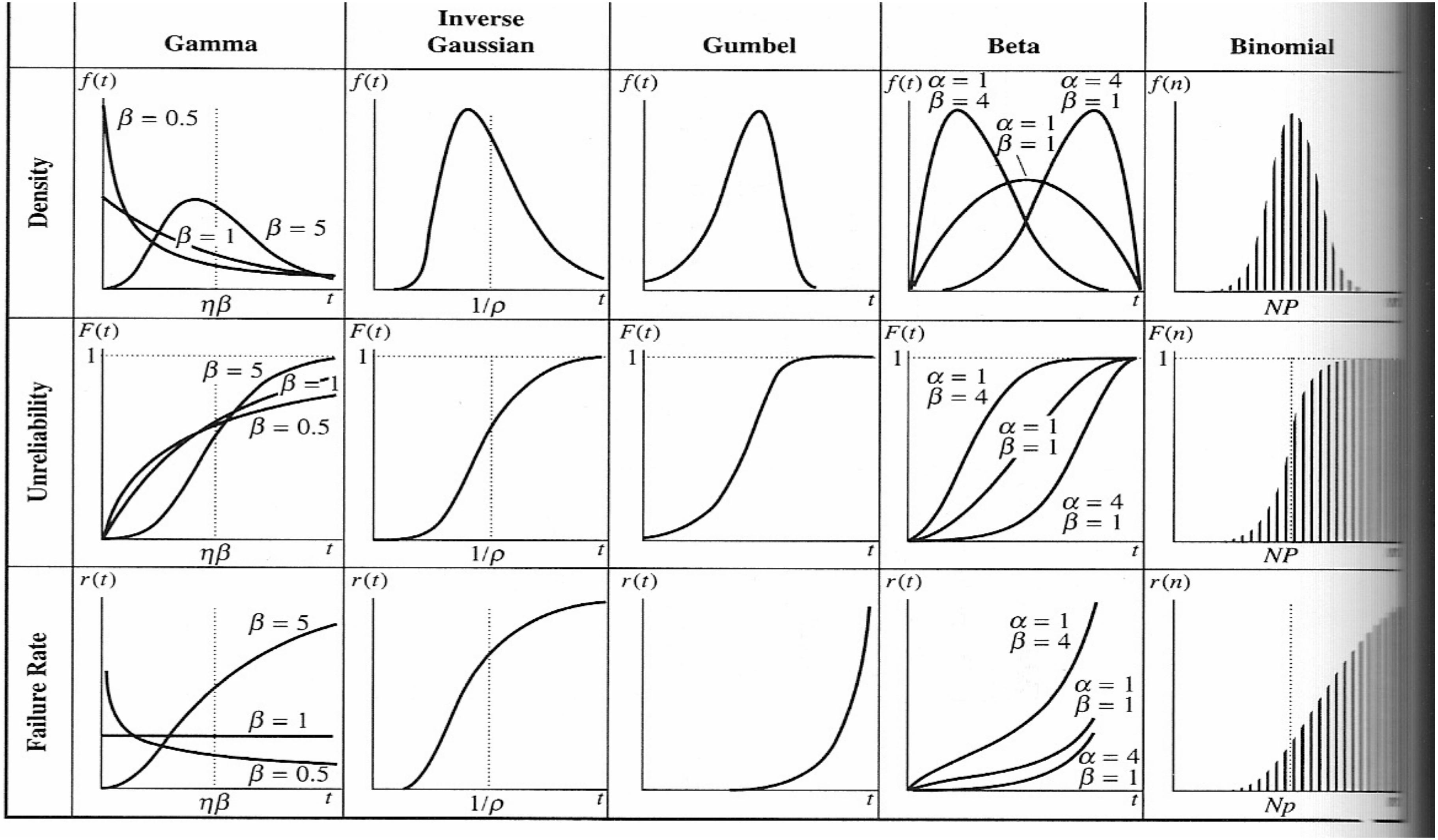
$$P_f = \int_0^t \lambda \cdot e^{-\lambda x} dx = 1 - e^{-\lambda t} \quad \text{where } \lambda = \text{constant failure rate.}$$

$$\text{Note: Probability of success} = P_s = 1 - P_f = e^{-\lambda t}$$

PDFs & CDFs of Typical Distributions



PDFs & CDFs of Typical Distributions cont.



Example of how Failure Characteristics of a Component is Determined

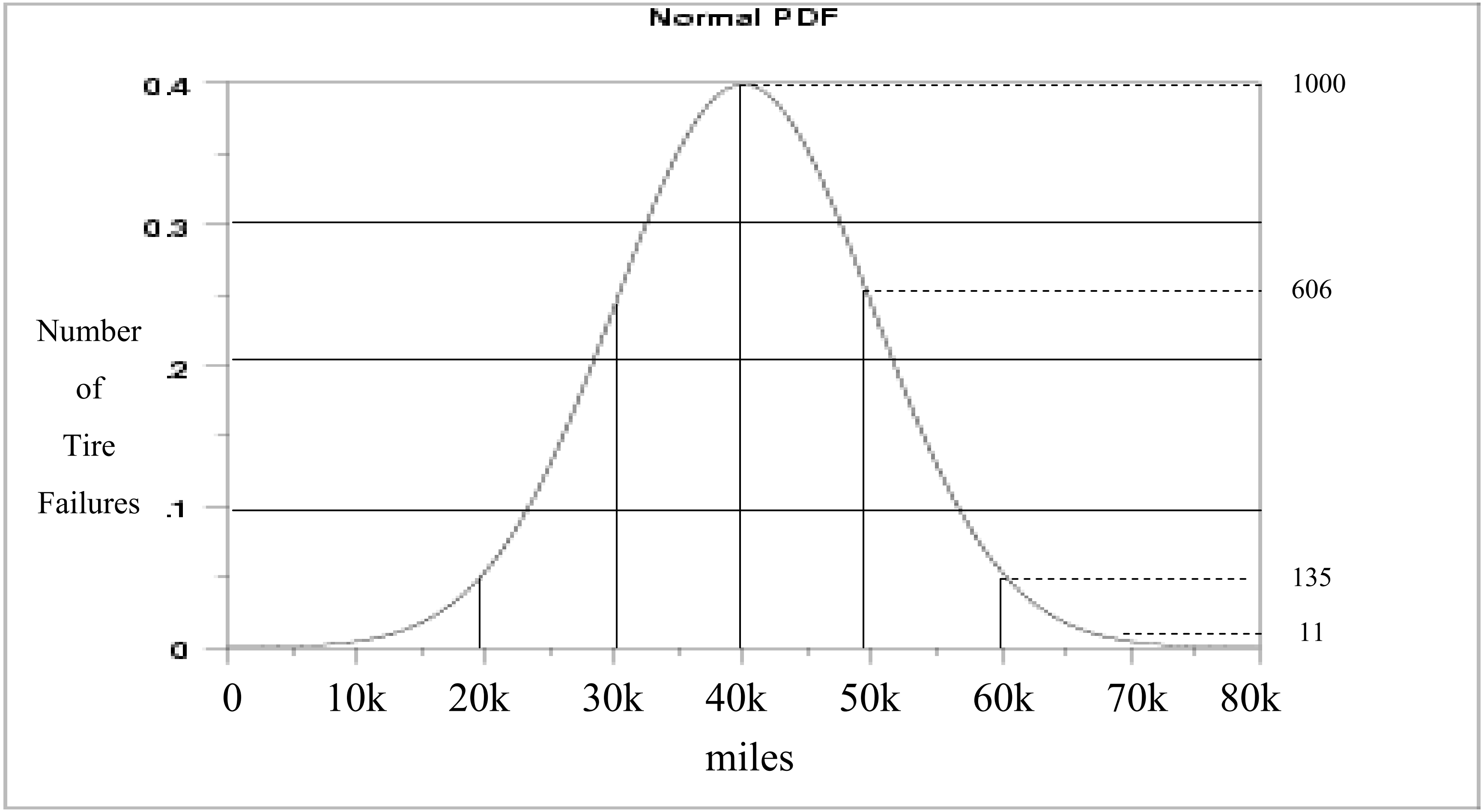
Hypothetical Construction of a PDF

Tire Failure Analysis (Normal Distribution):

Miles	# of Failures
0	0
10k	11
20k	135
30k	606
40k	1000
50k	606
60k	135
70k	11
80k	0
Total Failures	2504

Hypothetical Construction of a PDF

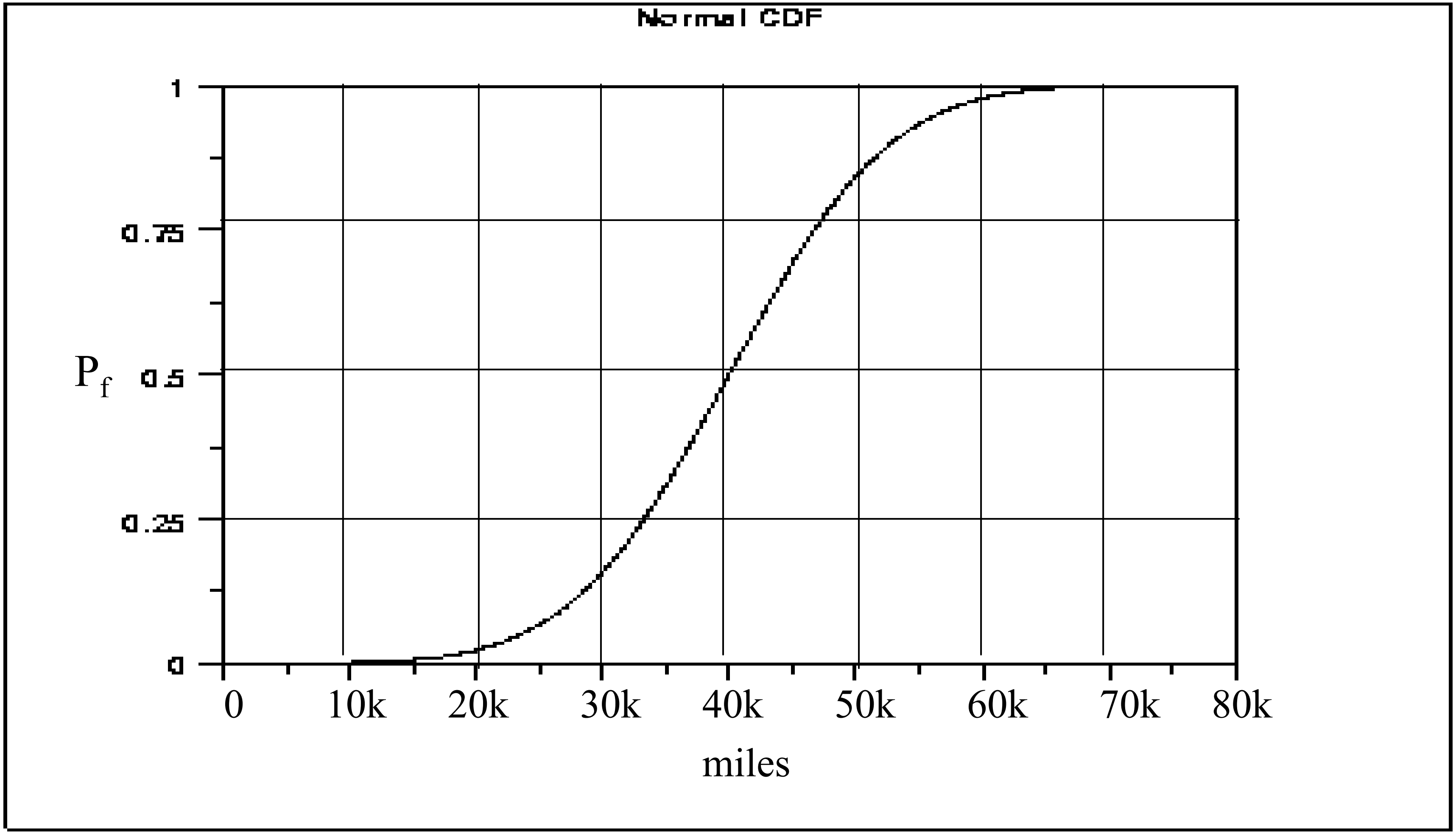
Tire Failure Analysis (Normal Distribution):



Note: Curve shown after normalization i.e. adjusted to set area under curve equal to 1.

Resultant CDF (P_f Curve) from PDF

Tire Failure Analysis (Normal Distribution):



Calculating Probability of Failure of a System

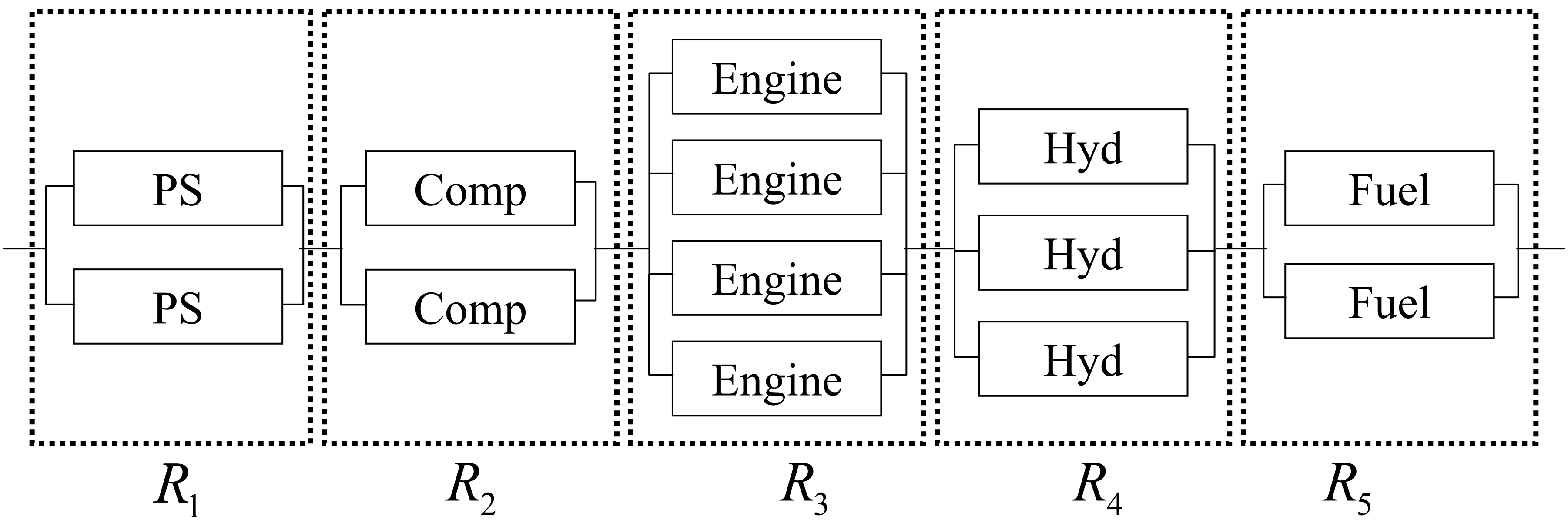
Calculating Probability of Failure of a System

System:

1) Develop Block (Reliability) Diagram – Show all series and redundant subsystems and/or black boxes

2) Given estimates of θ = MTBF or λ = failure rate, use binomial distribution to assess subsystem reliability. The assumption is that these estimates accurately represent the distribution parameter

Example: Truncated Aircraft System



Calculating Probability of Failure of a System cont.

(A) <u>Conditions and Assumptions</u>

- Time of Flight, t = 6 hours
- R(t) = Reliability of Each Subsystem
- (a)

Subsystem	Failure Rate	R(t)=p
Power Supply	0.001 failures/hr	0.994
Computer	0.015	0.914
Engine	0.004	0.976
Hydraulics	0.002	0.988
Fuel Distribution	0.003	0.982

- (b) Either power supply, computer, fuel system (1 out of 2) required for success
- (c) Any 3 out of the 4 engines required
- (d) Any 2 out of the 3 hydraulics required
- (e) Either one of the fuel systems required

Calculating Probability of Failure of a System cont.

(B) <u>Compute</u>

P.S.: P(2 or 1) = $p^2 + 2pq = (.994)^2 + 2(.994)(.006) = 0.999964 = R_1$

Comp: P(2 or 1) = $p^2 + 2pq = (.914)^2 + 2(.914)(.086) = 0.992604 = R_2$

Engines: P(4 or 3) = $p^4 + 4p^3q = (.976)^4 + 4(.976)^3(.024) = 0.996654 = R_3$

Hydraulics: P(3 or 2) = $p^3 + 3p^2q = (.988)^3 + 3(.988)^2(.012) = 0.999571 = R_4$

Fuel: P(2 or 1) = $p^2 + 2pq = (.982)^2 + 2(.982)(.018) = 0.999676 = R_5$

(C) P (System Success) = $\underset{i=1}{\overset{5}{\pi}} R_i = 0.9885$

(D) Effective MTBF: $R = e^{-\lambda t}$, $0.9885 = e^{-\lambda t}$

For t = 6, $\lambda = -\frac{1n(.9885)}{6} = 0.00193$

$\theta_E = 518$ hours

Calculating Probability of Failure of a System cont.

- A black box consists of 32 parts integrated circuits, wiring, boards, connectors, etc.
- The reliability engineer consults MIL-HDBK-217F to determine and/or compute the failure rates for each piece part. (Included in that handbook are qualified parts with their failure rates stipulated, as well as specific instructions for computing λ based on application of the part)
- Assuming that each of the 32 parts must function successfully (not fail) the task comes down to simply adding the failure rates.
- Thus for the black box not to fail in time t

$$R(\text{box}) = e^{-(\lambda_1 + \lambda_2 + \ldots \lambda_{32})t} = e^{-\lambda_E t}$$

λ_E= Effective failure rate of the box

Calculating Probability of Failure of a System cont.

For two boxes with failure rates respectively, the requirement that both boxes operate successfully for t hours is

$$R(t) = R_A(t) \cdot R_B(t)$$
$$= e^{-\lambda_A t} \cdot e^{-\lambda_B t}$$
$$= e^{-(\lambda_A + \lambda_B)t}$$

<u>Rule:</u> If n boxes are required to operate for a system to meet mission requirements then the system failure rate is the sum of the box failure rates

Since $$R_S = \prod_{i=1}^{n} R_i$$
$$= \prod_{i=1}^{n} e^{-\lambda_i t}$$
$$= e^{-\sum_{i=1}^{n} \lambda_i t} = e^{-t\sum_{i=1}^{n} \lambda i} = e^{-\lambda_s t}$$

Where $$\lambda_s = \sum_{i=1}^{n} \lambda_i$$ (System Failure Rate)

Derivation of MTBF

Computing MTBF

- Applicability

  The Exponential Distribution of times to failure has been proven to apply to electronic, electrical, and electromechanical systems, as well as complex systems including pneumatics, hydraulics.

  For the Exponential Distribution, Mean Time Between Failures

  MTBF (θ) is the inverse of Failure Rate (λ)

$$\theta = \frac{1}{\lambda}$$

- Reliability

  For units governed by the Exponential Function

  Either: $R = e^{-\lambda t}$

  Equivalency: $R = e^{\frac{-t}{\theta}}$

Computing MTBF

- What is the effective MTBF of a system consisting of 4 boxes, all of which must operate properly for the mission to succeed, given that the failure rates of the four boxes are $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. The system reliability, R_S, for this configuration is:

$$R_S = R_1 R_2 R_3 R_4$$

$$R_S = (e^{-\lambda_1 t})(e^{-\lambda_2 t})(e^{-\lambda_3 t})(e^{-\lambda_4 t})$$

$$R_S = e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)t}$$

And

$$R_S = e^{-\frac{t}{\theta_S}}$$

Therefore

$$\theta_S = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}$$

Thus <u>for a Series System</u>:

$$\lambda_s = \sum_{i=1}^{n} \lambda_1$$

$$\theta_s = \frac{1}{\lambda_s} = \frac{1}{\sum_{i=1}^{n} \lambda_i}$$

MTBF of 2 Boxes in Series (Exponential Distribution)

$$\text{Definition : System MTBF} = MTBF_S = \int_0^\infty R_S(t)\,dt$$

$$\lambda_S = \lambda_A + \lambda_B \Rightarrow R_S(t) = e^{-(\lambda_A + \lambda_B)t} \Rightarrow$$

$$MTBF_S = \int_0^\infty e^{-(\lambda_A + \lambda_B)t}\,dt \Rightarrow$$

$$MTBF_S = \frac{1}{\lambda_A + \lambda_B}$$

MTBF of 2 Boxes in Parallel (Exponential Distribution)

$$\text{Definition} \quad : MTBF_S = \int_0^{\infty} R_S(t)\,dt$$

$$R_S = 1 - (1 - R_A)(1 - R_B) = R_A + R_B - R_A R_B \Rightarrow$$

$$R_S(t) = e^{-\lambda_A t} + e^{-\lambda_B t} - e^{-(\lambda_A + \lambda_B)t} \Rightarrow$$

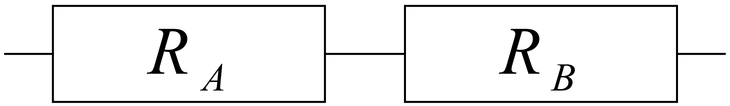
$$MTBF_S = \int_0^{\infty} e^{-\lambda_A t}\,dt + \int_0^{\infty} e^{-\lambda_B t}\,dt - \int_0^{\infty} e^{-(\lambda_A + \lambda_B)t}\,dt \Rightarrow$$

$$MTBF_S = \frac{1}{\lambda_A} + \frac{1}{\lambda_B} - \frac{1}{\lambda_A + \lambda_B}$$

$$\text{Note} \;:\; \lambda_A = \lambda_B \Rightarrow MTBF_S = \frac{2}{\lambda_A} - \frac{1}{2\lambda_A} = \frac{3}{2\lambda_A}$$

Summary of Reliability of Series & Parallel Circuits

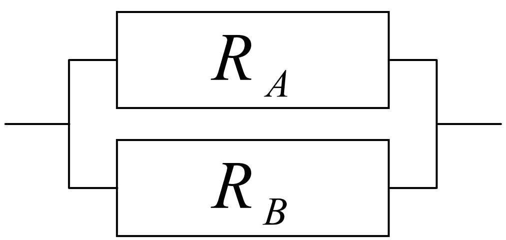
1. Series



$$R_s = R_A \cdot R_B = e^{-(\lambda_A + \lambda_B)t} \qquad \lambda_s = \lambda_A + \lambda_B$$

2. Parallel (refer to page on MTBF of 2 boxes in Parallel)

Box Reliability not Equal

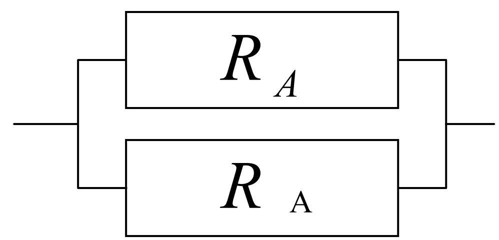


$$R_s = 1 - \left[(1 - R_A)(1 - R_B)\right]$$

Using MTBF is derived on previous page:

$$\lambda_s = \frac{\lambda_A \lambda_B (\lambda_A + \lambda_B)}{(\lambda_A + \lambda_B)^2 - \lambda_A \lambda_B}$$

Box Reliability Equal

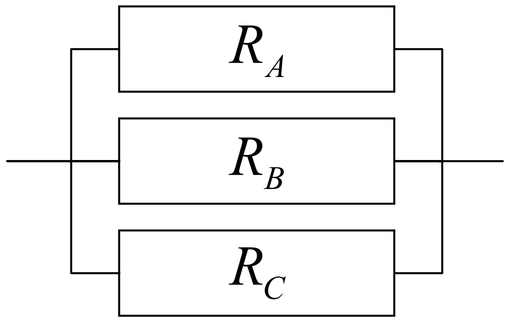


$$R_s = 1 - \left[(1 - R_A)^2\right]$$

$$\lambda_s = \frac{2\lambda_A^3}{3\lambda_A^2} = \frac{2\lambda_A}{3}$$

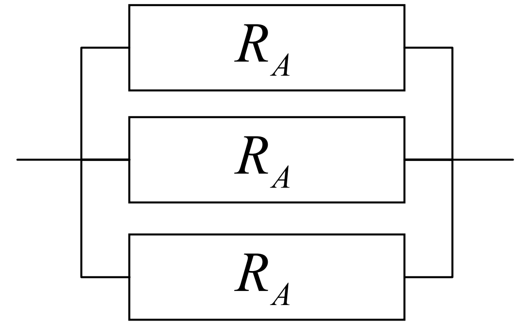
Summary of Reliability of Series & Parallel Circuits

4.



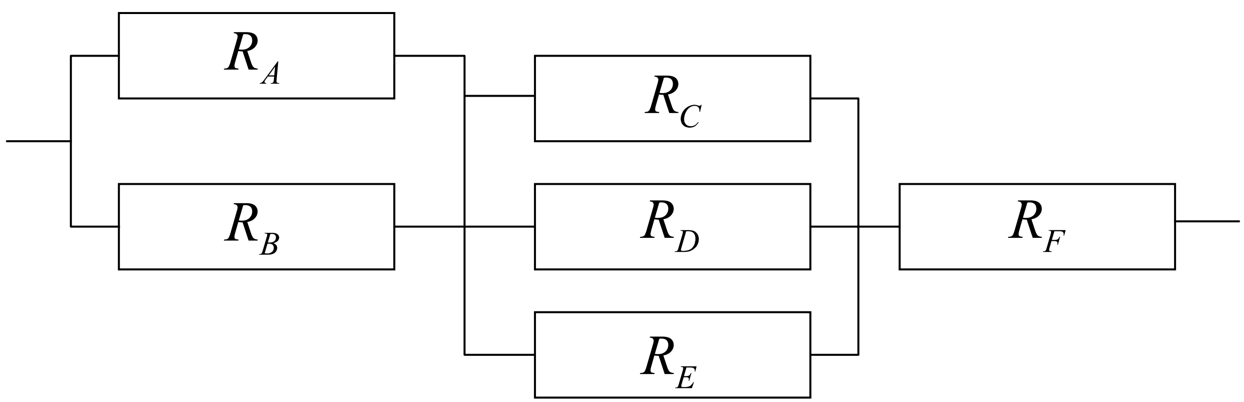
$$R_s = 1 - (1 - R_A)(1 - R_B)(1 - R_C)$$

5.



$$R_s = 1 - [(1 - R_A)^3]$$

6.



$$R_s = [1 - (1 - R_A)(1 - R_B)][1 - (1 - R_C)(1 - R_D)(1 - R_E)][1 - (1 - R_F)]$$

Why Weibull Analysis?

Weibull Analysis

Why Weibull?

- Primary advantage is the ability to provide very accurate failure analysis and failure forecasting with extremely small sample size, resulting in savings in time and money.
- Weibull distributions include a large variety of distribution shapes which can be used to best fit life data. This process is known as curve fitting
- Weibull plots support Maintenance tasks, particularly Reliability centered Maintenance.
- Weibull analysis reduces complicated mathematical integrals to simpler algebraic equations thereby greatly simplifying probability of failure computations

Weibull cont.

It is commonly known that the Weibull equation

$$F(t) = 1 - e^{-\left[\left(\frac{t-b}{a}\right)^{c}\right]}$$

can be used to curve fit (approximate) the P_f of components that exhibit non-constant failure rates. The obvious questions that arise are :

1. How is it done? and
2. How accurate are the approximations?

Weibull cont.

Example:

A certain mechanical device has exhibits a normal failure distribution with u = 100, s = 20, and hl (hours already logged) = 5.

Set a = 69.62, b = 32.384, c = 3.456, then

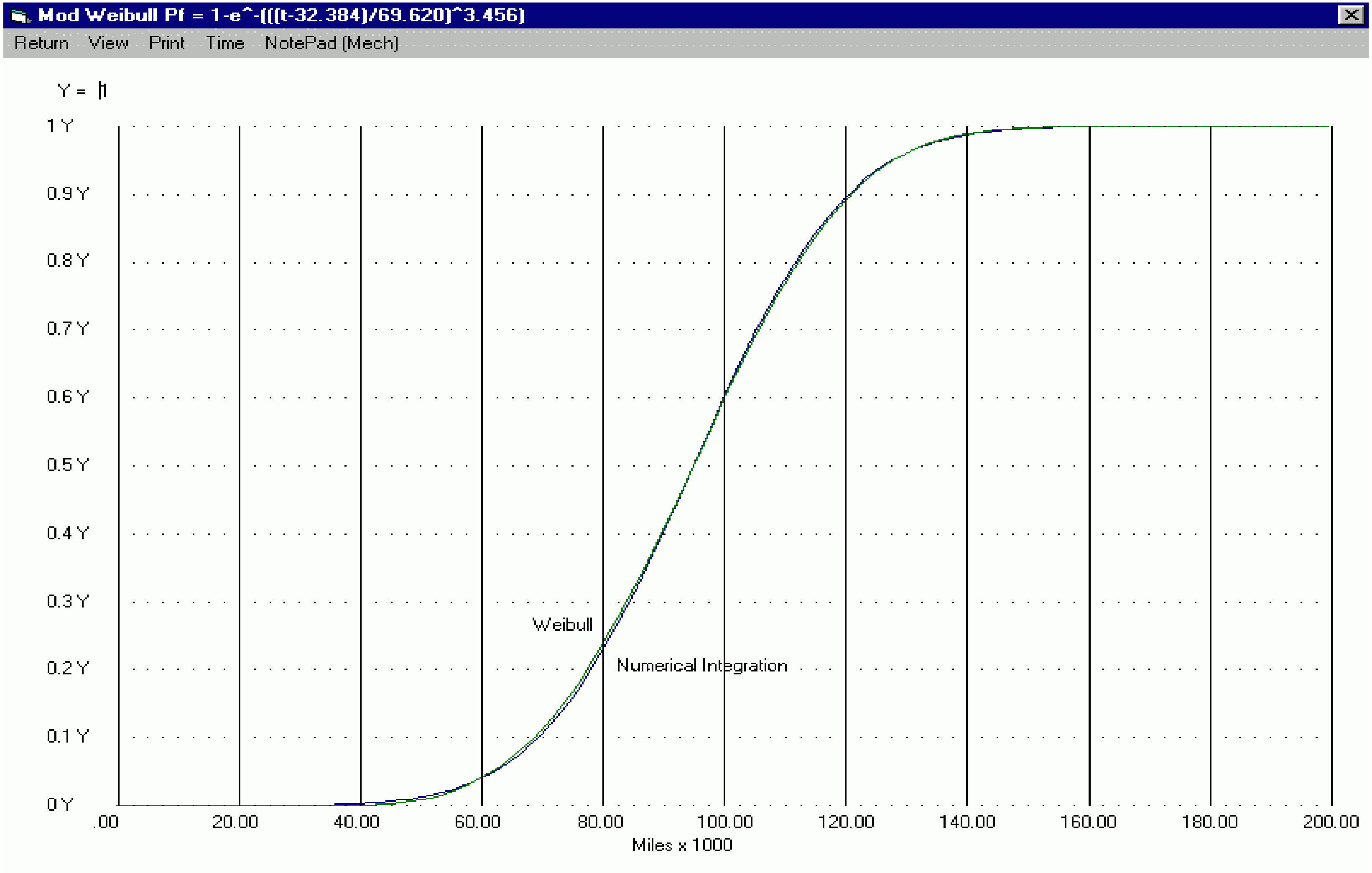
$$P_f = \frac{1}{s\sqrt{2\pi}} \int_0^t e^{-\left(\frac{(x-u+hl)^2}{2s^2}\right)} dx \approx 1 - e^{-\left[\left(\frac{t-b}{a}\right)^c\right]}$$

Note: The derivation of a, b, and c is a subject for another paper.

Notice the correlation shown in the graph that follows:

Weibull cont. (How accurate are these approximations?)

Pf vs. Miles (0 to 200k miles)

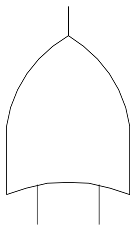


Weibull cont. (Mechanical Device in Series with an Electrical Device)

A simple system is made up of a mechanical device with a normal distribution of failure, in series with an electrical device. The mechanical device has u = 100, s = 20, with k = 10 hours logged. The electrical device has a failure rate λ of .01 failures per hour. Calculate the P_f of the system.

Logic Diagram

$P_f = F(t) + G(t) - F(t)\,G(t)$



F(t) G(t)

$$F(t) = P_f\ (\text{mech}) = 1 - e^{-\left[\left(\frac{t-b}{a}\right)^{c}\right]}$$

where $a = 3.481s$, and $b = u - k - a\,[-\ln(0.5)]^{\frac{1}{c}}$

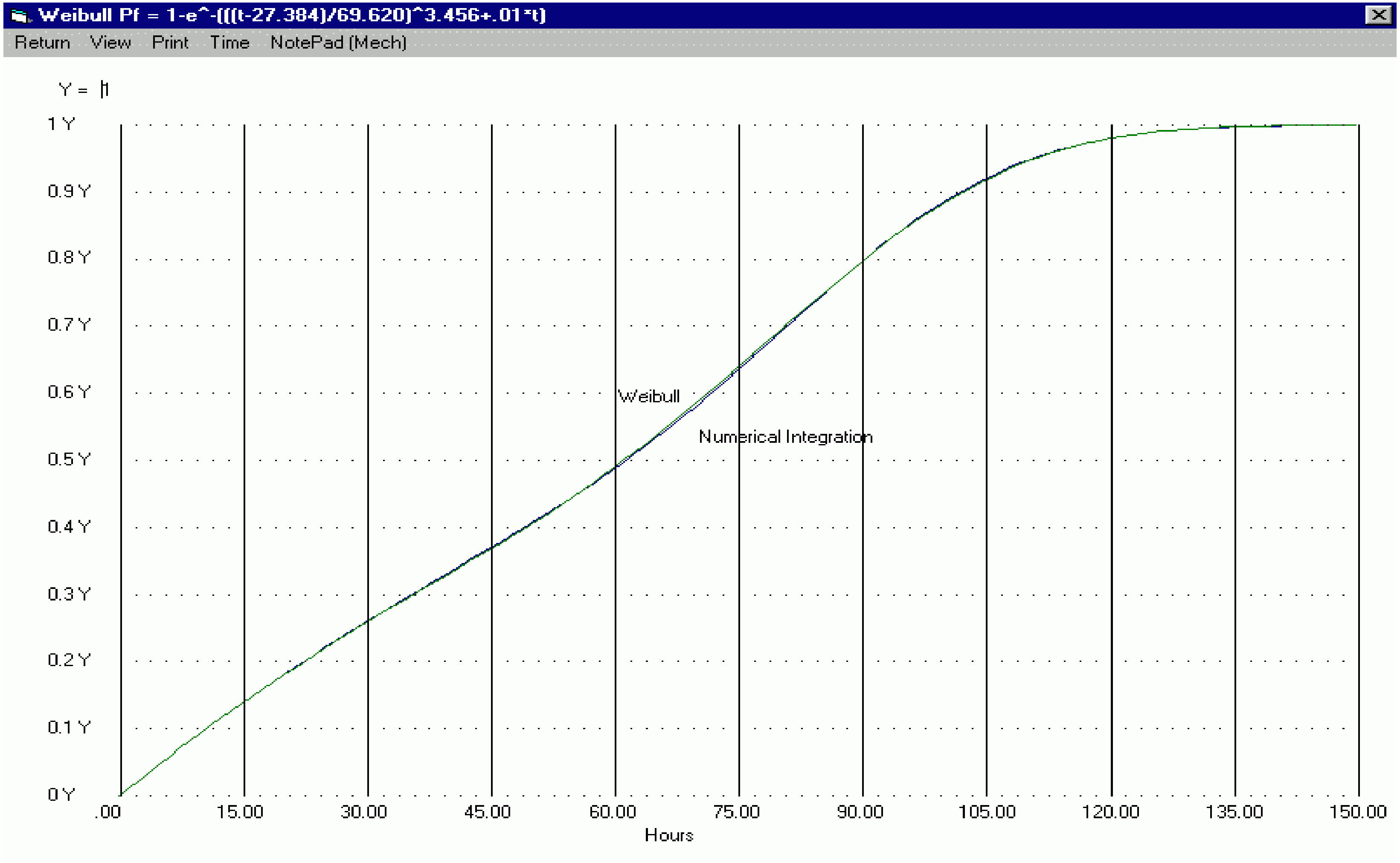
$$G(t) = P_f\ (\text{elect}) = 1 - e^{-\lambda t} \quad \text{where} \quad \lambda = .01$$

$$\text{Now } P_f = F(t) + G(t) - F(t)G(t) = 1 - e^{-\left[\left(\frac{t-b}{a}\right)^{c} + \lambda t\right]} \text{ and}$$

$$u = 100,\ s = 20,\ k = 10,\ \lambda = .01 \Rightarrow P_f = 1 - e^{-\left[\left(\frac{t-27.384}{69.620}\right)^{3.456} + .01t\right]}$$

Weibull cont. (Graph of Mechanical in Series with an Electrical Device)

Pf vs. Hours **(0 to 150 hours)**



Reliability vs. Safety

Reliability vs. Safety

- Reliability and Safety should not be equated (generally true)
- Improved system reliability does not necessarily improve system safety

  - Axioms: Adding series components always reduces system reliability

    Adding parallel capabilities always improves system reliability

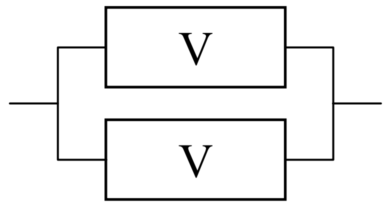
  - Not always true for system safety

- Each configuration contingency depends on failure mode hypothesized

Reliability vs. Safety (Example)

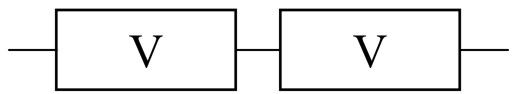
- Fuel Valves (ascent and descent engines)

Reliability



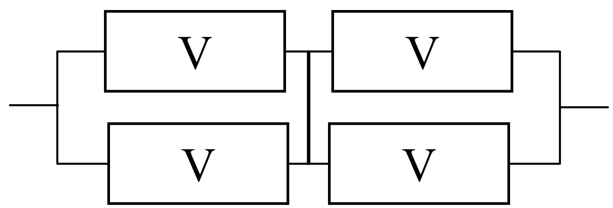
To Assure Operation (A)

Safety



To Preclude Leakage (B)

Solution



(C)

No single failure (inability for valve to open, valve leakage)

- <u>Configurations Applicable to Commercial Aircraft</u>

Points to Keep in Mind

- Reliability = Probability of success
- Distinction should be made between constant and non-constant failure rate devices. Mechanical devices exhibit non-constant failure rates.
- Distinction should be made between combinatorial and non-combinatorial logic when performing system failure analysis.
- Non-combinatorial logic cannot be expressed using logic gates.
- Mil-Hdbk 217 out of date.
- Math modeling of failure characteristics of components involves physics.
- Math modeling of failure characteristics of a system is all math
- Markov is a buzz word for methods used to solve non-combinatorial problems.
- Several Reliability SW packages are out on the market advertising Markov Analysis. Who or what organization is validating them?