Why use a Spares Calculator?

- To calculate number of spares required for a single unit to operate successfully for a given time interval, and for a given % confidence.
- Mathematics involved is non-trivial.
- Spares calculations without a computer program are prone to error due to multiple steps involved.
- Time required for calculations greatly reduced.

Summary of Techniques Used in Spares Calculation

Poisson Cumulative Probability Distribution

Stated mathematically:

$$P(k, n\lambda t) = \sum_{j=0}^{k} \frac{e^{-n\lambda t} \cdot (n\lambda t)^{j}}{j!} \geq C$$

Where C = confidence level

k = recommended spares quantity

n = number of items deployed

λ = predicted failure rate of item

t = operating time of period of interest

Binomial Probability Distribution

In order to understand the Poisson Distribution more clearly, we will discuss the Binomial Distribution which is more widely known, and somewhat easier to understand, and then compare the two.

Let p + q = 1 then

$$(p+q)^n = \sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!}\, p^{n-k} \cdot q^k = 1$$

Example: Let p = 0.9, n = 3

$$(.9+.1)^3 = (.9)^3 + 3(.9)^2(.1) + 3(.9)(.1)^2 + (.1)^3$$
$$= 0.729 + 0.243 + 0.027 + 0.001 = 1$$

Simply algebraic multiplication will prove the above.

Binomial Distribution (Pascal's Triangle)

Simple way to calculate $$\frac{n!}{(n-k)!\cdot k!} = \mathrm{cmb}(n,\ k)$$

where cmb(n, k) = number of ways k items can be selected from n items.

n	n! / (n-k)!k!
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1

With respect to the triangle, n = row number and k = position number starting with k = 0.

Example cmb(7, 2) = 7! / 5!·2! = 21.

Notes:

1. known as combinatorial logic
2. cmb(n, 0) = 1 and cmb(n, n) = 1
3. cmb short for combinations

Binomial Probability Distribution cont.

Example:

Three black boxes are operating "Active Redundant". What is the probability that exactly one black box will fail if the reliability (probability of success) of each box is 0.9?

$$1 = (p + q)^3$$

$$1 = p^3 + 3p^2q + 3pq^2 + q^3$$

$$1 = (.9)^3 + 3(.9)^2(.1) + 3(.9)(.1)^2 + (.1)^3$$

P(exactly 0 failures)

P(exactly 1 failure)

P(exactly 2 failures)

P(exactly 3 failures)

$$\text{Answer} : 3(.9)^2(.1) = 0.243$$

Binomial Cumulative Probability Distribution

Stated mathematically:

$$P(n,k) = \sum_{j=0}^{k} \frac{n!}{j!(n-j)!}\, p^{n-j} \cdot (1-p)^{j} = \sum_{j=0}^{k} \mathrm{cmb}(n,j) \cdot p^{n-j} \cdot (1-p)^{j}$$

Where n = number of items (sample size)
p = probability of success of each item
P(n, k) = probability of at least n-k successes

Example:
Five black boxes are operating "Active Redundant". What is the probability that <u>at least three</u> black boxes will operate if the reliability (probability of success) of each box is 0.9?

$$\text{Answer} : P(5,\ 2) = (.9)^{5} + 5(.9)^{4}(.1) + 10(.9)^{3}(.1)^{2}$$

exactly 5 successes
exactly 4 successes
exactly 3 successes

Poisson Probability Distribution

Stated mathematically:

$$P(k, nq) = \frac{e^{-nq} \cdot (nq)^k}{k!}$$

Where n = number of items (sample size)

q = probability of failure of one item

k = exact number of failed items

P(k, nq) = probability of <u>exactly</u> k failures

Probability of exactly k Failures (Poisson vs. Binomial)

n = 6 items, k = # of failures, q = 0.1, p = 1 - q

	Poisson	Binomial	
k	$\dfrac{(nq)^k \cdot e^{-nq}}{k!}$	$cmb(n,\ k) \cdot p^{n-k} \cdot q^k$	$cmb(n,\ k)$
0	0.548811	0.531441	1
1	0.329286	0.354294	6
2	0.098786	0.098415	15
3	0.019757	0.01458	20
4	0.002963	0.001215	15
5	0.000355	0.000054	6
6	0.000035	0.000001	1

Poisson vs. Binomial Cumulative Distribution

n = 6 items,   k = # of failures,   q = 0.1,   p = 1 - q

	Poisson	Binomial	Interpretation
k	$\sum_{j=0}^{k} \frac{(nq)^{j} \cdot e^{-nq}}{j!}$	$\sum_{j=0}^{k} \mathrm{cmb}(n, j) \cdot p^{n-j} \cdot q^{j}$	
0	0.548811	0.531441	P(0 failures)
1	0.878098	0.885735	P(1 or less failures)
2	0.976884	0.98415	P(2 or less failures)
3	0.996641	0.99873	P(3 or less failures)
4	0.999605	0.999945	P(4 or less failures)
5	0.999961	0.999999	P(5 or less failures)
6	1	1	P(6 or less failures)

Sample Spares Problem

Four black boxes are operating “Active Redundant”. What are the probabilities that all four black boxes will operate with 0, 1, 2 ,3, and 4 spares if the reliability of each box is 0.9 for a given period of time?

$$1 = (p + q)^4$$

$$1 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$$

$$1 = (.9)^4 + 4(.9)^3(.1) + 6(.9)^2(.1)^2 + 4(.9)(.1)^3 + (.1)^4$$

$$\text{P(0 spares)} = \text{First term} = (.9)^4$$

$$\text{P(1 spare)} = \text{Sum of first 2 terms} = (.9)^4 + 4(.9)^3(.1)$$

$$\text{P(2 spares)} = \text{Sum of first 3 terms} = (.9)^4 + 4(.9)^3(.1) + 6(.9)^2(.1)^2$$

$$\text{P(3 spares)} = \text{Sum of first 4 terms}$$

$$\text{P(4 spares)} = \text{Sum of all terms} = 1$$

Poisson vs. Binomial Probability Distribution

$q = 0.1,\ \ p = 0.9$

$\dfrac{e^{-4q}}{1}$	$\dfrac{(4q)^1 \cdot e^{-4q}}{1}$	$\dfrac{(4q)^2 \cdot e^{-4q}}{2}$	$\dfrac{(4q)^3 \cdot e^{-4q}}{6}$	$\dfrac{(4q)^4 \cdot e^{-4q}}{24}$
0.67032	0.26812	0.05362	0.00715	0.00071
p^4	$4p^3 \cdot q$	$6p^2 \cdot q^2$	$4p \cdot q^3$	q^4
0.6561	0.2916	0.0486	0.0036	0.0001