# Definite Integrals

Theorem:

$$\int_0^{na} x^r \cdot e^{-\frac{x}{n}}\, dx \quad = \quad n^{r+1}\int_0^{a} x^r \cdot e^{-x}\, dx$$

Proof :

$$\text{Left side} = \int_0^{na} x^r \cdot \left(1 - \frac{x}{n} + \frac{x^2}{n^2 \cdot 2!} - \frac{x^3}{n^3 \cdot 3!} + \cdots\right) dx = \int_0^{na} \left( x^r - \frac{x^{r+1}}{n} + \frac{x^{r+2}}{n^2 \cdot 2!} - \frac{x^{r+3}}{n^3 \cdot 3!} + \cdots \right) dx$$

$$= \Big|_0^{na} \frac{x^{r+1}}{(r+1)} - \frac{x^{r+2}}{n(r+2)} + \frac{x^{r+3}}{n^2(r+3)\cdot 2!} - \frac{x^{r+4}}{n^3(r+4)\cdot 3!} + \cdots$$

$$= \frac{(na)^{r+1}}{(r+1)} - \frac{(na)^{r+2}}{n(r+2)} + \frac{(na)^{r+3}}{n^2(r+3)\cdot 2!} - \frac{(na)^{r+4}}{n^3(r+4)\cdot 3!} + \cdots$$

$$\therefore \text{ Left side} = \frac{n^{r+1}a^{r+1}}{(r+1)} - \frac{n^{r+1}a^{r+2}}{(r+2)} + \frac{n^{r+1}a^{r+3}}{(r+3)\cdot 2!} - \frac{n^{r+1}a^{r+4}}{(r+4)\cdot 3!} + \cdots$$

$$\text{Right side} = n^{r+1}\int_0^{a} x^r \cdot \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right) dx = n^{r+1}\int_0^{a} \left(x^r - x^{r+1} + \frac{x^{r+2}}{2!} - \frac{x^{r+3}}{3!} + \cdots\right) dx$$

$$= n^{r+1}\Big|_0^{a} \frac{x^{r+1}}{(r+1)} - \frac{x^{r+2}}{(r+2)} + \frac{x^{r+3}}{(r+3)\cdot 2!} - \frac{x^{r+4}}{(r+4)\cdot 3!} + \cdots$$

$$= n^{r+1}\left[ \frac{a^{r+1}}{(r+1)} - \frac{a^{r+2}}{(r+2)} + \frac{a^{r+3}}{(r+3)\cdot 2!} - \frac{a^{r+4}}{(r+4)\cdot 3!} + \cdots \right]$$

$$\therefore \text{ Right side} = \frac{n^{r+1}a^{r+1}}{(r+1)} - \frac{n^{r+1}a^{r+2}}{(r+2)} + \frac{n^{r+1}a^{r+3}}{(r+3)\cdot 2!} - \frac{n^{r+1}a^{r+4}}{(r+4)\cdot 3!} + \cdots \Rightarrow$$

Left side $=$ Right side //