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Calculating Failure Rates of Series / Parallel Networks

By

Introduction

Recently, the author attempted to calculate the failure rate (FR) of a series/parallel (active redundant, without repair) reliability network using the Reliability Toolkit: Commercial Practices Edition published by the System Reliability Center (SRC) as a guide. The Toolkit's approach for FR calculation for a single branch seemed to be very thorough. So the FR for each individual branch was calculated. Since several branches were in series, the FRs of each branch were then added together. Closer examination revealed that this approach was an oversimplification and did not take into account all possible combinations (ways) that individual components could fail. A closer review of the Toolkit revealed that FR calculations of single branches with n components in parallel were indeed treated very thoroughly. However, the Toolkit lacks detail in describing a method for handling multiple branches in series

A quick review of the software QuART Pro Ver. 2.0 Release 1 Build 70 was performed. It also seemed to deal with single branches very thoroughly but not multiple branches in series.

Objectives

The objectives of this article are to:

- Describe two erroneous approaches commonly performed when calculating FR of Serial/Parallel reliability networks.
- Provide an example of a correct approach.
- Approximate the percent errors one can expect when FR is calculated erroneously.

Nature of the Problem:

System Reliability is calculated as a combination of series and parallel paths and can be expressed as a failure rate. Calculating the FR of a series network as shown in Figure 1 is a simple act of just adding all of the FRs in the series string together, and should need no further explanation.

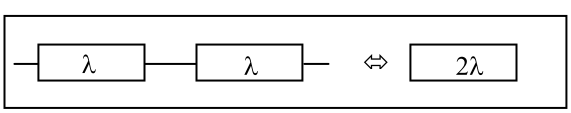

Figure 1. Series network

However, calculating the Reliability and/or FR of parallel networks requires a little more work. The Toolkit contains excellent information for doing this. See Toolkit Table 6.2-2 for calculating Reliability, and Toolkit Table 6.2-3 for calculating FR for parallel networks.

For example, consider the network in Figure 2.

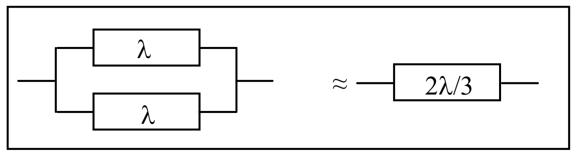

Figure 2. Parallel network

From Table 6.2-2 we get $R(t) = 2e^{-\lambda t} - e^{-2\lambda t}$ and from Table 6.2-3 (equation 4) we get

$$FR = \frac{\lambda}{1/1 + 1/2} = \frac{2\lambda}{3}$$

For the network in Figure 3, first collect (add) all lambdas in series as shown, and then from the Toolkit tables get:

$$R(t) = 2e^{-2\lambda t} - e^{-4\lambda t} \text{ and } FR = \frac{2\lambda}{1/1 + 1/2} = \frac{4\lambda}{3}$$

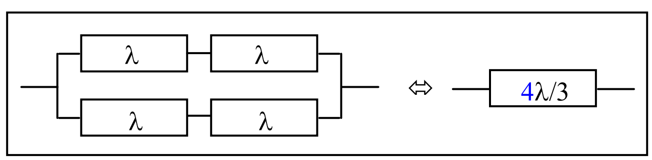

Figure 3. Network of series elements in parallel

The tables provide correct solutions for the networks in Figures 1, 2, and 3. However, a potential problem occurs when calculating the FR of a series/parallel network as shown in Figure 4. Analysts commit a very common error by intuitively calculating the FR of each parallel branch first, then add each branch FR together, since the branches are in series, and erroneously calculate FR = 4λ/3 as in this example. This FR calculation actually correlates to that for the network in Figure 3. It is very important to understand that the network in Figure 3 and the network in Figure 4 are not equivalent.

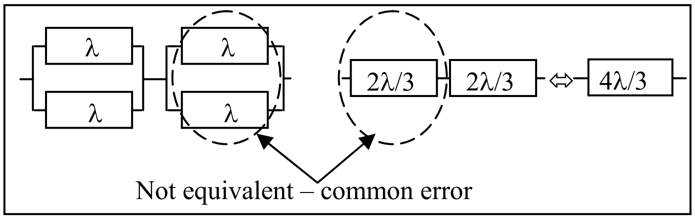

Figure 4. Series-parallel network

Root Cause of Problem

The correct approach to calculate the FR of the network in Figure 4 (or any other network for that matter) is to calculate the reliability of each branch first, then multiply together the reliability of each branch. Assume the reliability of Branch 1 is $R_1(t)$, the reliability of Branch 2 is $R_2(t)$, etc. Then network reliability = $R(t) = R_1(t) \cdot R_2(t) \cdots R_n(t)$. It is important to note that the reliability of each branch $R_i(t)$ must be kept in terms of the failure rates of the components within the branch, and **not** in terms of the failure rate of the branch itself. Therein lies the root cause of the problem. The network FR can then be computed using the following definition.

$$FR = \frac{1}{MTTF} = 1/\int_0^\infty R(t)\,dt$$

Correct Approach (Network in Figure 4)

From the Toolkit Table 6.2-2, the reliability of each branch is:

$$2e^{-\lambda t} - e^{-2\lambda t} \Rightarrow$$

$$R(t) = (2e^{-\lambda t} - e^{-2\lambda t})(2e^{-\lambda t} - e^{-2\lambda t}) = 4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t} \Rightarrow$$

$$MTTF = \int_0^\infty R(t)dt = \int_0^\infty (4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t})dt \Rightarrow$$

$$MTTF = \frac{4}{2\lambda} - \frac{4}{3\lambda} + \frac{1}{4\lambda} = \frac{11}{12\lambda} \Rightarrow \text{True FR} = \frac{12}{11}\lambda$$

Incorrect Approach A (Network in Fig. 4)

From the Toolkit Table 6.2-3, the FR of the each branch is 2λ/3. It is intuitive to add these failures rates since the two branches are in series. This erroneous approach yields 2λ/3 + 2λ/3 = 4λ/3 which is obviously not equal to 12λ/11. This approach will yield an approximate 22% error.

Incorrect Approach B (Network in Fig. 4)

Another erroneous approach is to try to calculate FR as a function of time. For example, given that t = 10 hours, and λ = 250 fpmh (failures per million hours), one may be tempted to calculate network FR as follows:

$$R(10) = 4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t} = 4e^{-2*250*10/10^6} - 4e^{-3*250*10/10^6} + e^{-4*250*10/10^6} = 0.99998753$$

Then using FR = –ln(R(t))/t:

$$FR = -\ln(0.99998753)/10 = 1.246 \times 10^{-6} = 1.246 \text{ fpmh}$$

This does not equal 12λ/11 = 12*250/11 = 273 fpmh.

Note that 1.246 fpmh is only an "apparent" FR measured during a period 10 hours, not to be confused with the FR as formally defined previously.

Given t = 100 hours, then:

$$R(100) = 4e^{-2*250*100/10^6} - 4e^{-3*250*100/10^6} + e^{-4*250*100/10^6} = 0.99878117 \Rightarrow$$

$$FR = -\ln(0.99878117)/100 = 12.196 \times 10^{-6} = 12.196 \text{ fpmh}.$$

Note that 12.196 fpmh is another "apparent" FR measured at 100 hours. Notice also that the value of the "apparent" FR will vary with t.

Networks with all components having the same lambda are not very common. An example of the correct approach on a more practical (common) network is shown next.

A Correct Approach for Calculating Network Failure Rate

Consider the network shown in Figure 5 with failure rates a, b, and c. The definition of success for the network, is defined as at least 1 of 2 components of the left branch, and at least 2 of 3 components of the middle branch must be functional. From the Toolkit Table 6.2-2, the reliability of the left branch is $2e^{-at} - e^{-2at}$, and the middle branch is $3e^{-2bt} - 2e^{-3bt}$. By definition, the reliability of the right branch is e^{-ct}. Network reliability R(t) is calculated by multiplying the three branch reliabilities together.

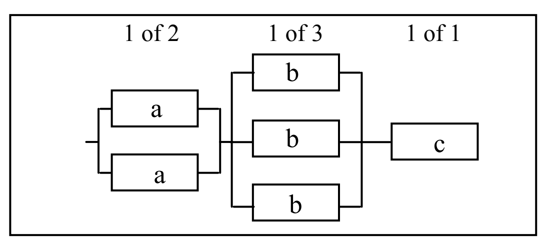

Figure 5. Example with multiple paths.

Therefore:

$$R(t) = (2e^{-at} - e^{-2at})(3e^{-2bt} - 2e^{-3bt}) \cdot e^{-ct}$$
$$= 6e^{-(a+2b+c)t} - 4e^{-(a+3b+c)t} - 3e^{-(2a+2b+c)t} + 2e^{-(2a+3b+c)t}$$

and

$$MTTF = \int_0^{\infty} R(t)dt$$

$$= \int_0^{\infty} (6e^{-(a+2b+c)t} - 4e^{-(a+3b+c)t} - 3e^{-(2a+2b+c)t} + 2e^{-(2a+3b+c)t})dt$$

$$MTTF = \frac{6}{a+2b+c} - \frac{4}{a+3b+c} - \frac{3}{2a+2b+c} + \frac{2}{2a+3b+c}$$

The rest is algebra. Calculate MTTF using known values of a, b, and c, then take the reciprocal since $FR = 1/MTTF$

Erroneous Method A

A common error is performed when the analyst calculates the FR of each individual branch first, then adds all calculated branch FRs together. Note in the previous example, the FR of the left branch is 2a/3, FR of the middle branch is 6b/5, and the FR of the right branch is c.

$$\text{Therefore } FR(\text{erroneous}) = \frac{2a}{3} + \frac{6b}{5} + c = \frac{10a+18b+15c}{15}$$

Simple algebra shows that:

$$\frac{10a+18b+15c}{15}$$

$\Rightarrow$ is not equal to:

$$\frac{1}{\frac{6}{a+2b+c} - \frac{4}{a+3b+c} - \frac{3}{2a+2b+c} + \frac{2}{2a+3b+c}}$$

Error Magnitude Estimation for Erroneous Approach A

Five sample networks were chosen starting with a 2 row by 2 column network as shown in the table below. For the sake of simplicity, all network components were assigned the same lambda. In each case, the True FR was compared with the FR calculated erroneously by simply adding FRs of each branch. The % error was then measured. From the table, it can be easily seen, that the larger the network, the larger the error.

TABLE 1: Error Magnitude Estimation Table for Erroneous Approach A

Network Configuration (Rows x Columns)	**True FR**	**Erroneous FR** (adding FRs of each Branch)	**% Error**
2x2	$\frac{12}{11}\lambda$	$\frac{2\lambda}{3}+\frac{2\lambda}{3}=\frac{4}{3}\lambda$	22
2x3	$\frac{10}{7}\lambda$	$\frac{2\lambda}{3}+\frac{2\lambda}{3}+\frac{2\lambda}{3}=\frac{6}{3}\lambda$	40
2x4	$\frac{280}{163}\lambda$	$\frac{2\lambda}{3}+\frac{2\lambda}{3}+\frac{2\lambda}{3}+\frac{2\lambda}{3}=\frac{8}{3}\lambda$	55
3x2	$\frac{60}{73}\lambda$	$\frac{6\lambda}{11}+\frac{6\lambda}{11}=\frac{12}{11}\lambda$	32
3x3	$\frac{2520}{2467}\lambda$	$\frac{6\lambda}{11}+\frac{6\lambda}{11}+\frac{6\lambda}{11}=\frac{18}{11}\lambda$	60

Error Magnitude Estimation for Erroneous Approach B

The error magnitude for this approach will depend on the chosen value of t, and would be very difficult to express as an equation. Suffice to say that the FR calculated by this approach may not come close, or even resemble the correct result.

Conclusions

Calculating the failure rate (FR) of a series/parallel (active redundant, without repair) reliability network is not as simple as one might believe, an incorrect approach can lead to subtle but substantial errors. Closer examination reveals that one must carefully account for all possible paths of success for multiple networks having branches in series.

In general, the larger the network, the larger the potential error when oversimplified approaches are used in calculating the reliability of these complex networks. The percent error, although not proven here, is a function of network size, network configuration, values of lambdas, and in some cases, a function of time.

Reference

Reliability Engineering, ARINC Research Company, Michael Pecht, Editor, Prentice-Hall Inc, pages 202 to 226.